Simulation-Based Evolutionary Optimization of Complex Multi-Location Inventory Models

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***Abstract*—A common strategy to solve real-world problems in economics that cannot be solved analytically is the simulation-based optimization approach, which requires the complete simulation of the system to evaluate its configuration. This idea is illustrated for a very general class of logistics systems, represented by multi-location inventory models with lateral transshipments, applying evolutionary optimization. These systems have become increasingly important in a globalized economic setting and strategies for the control of transshipments are questions of current research. In particular, we extend existing approaches by considering the future behavior of the system in order to reallocate or to increase assets.**

***Index Terms*—Simulation-based optimization, inventory models, lateral transshipments, evolutionary optimization.**

I. Introduction

Multi-location inventory models with lateral transshipments (MLIMT) model the following situation. A given number of locations have to meet a demand for some products during a defined planning horizon. Each location may obtain new product units either by ordering from an outside supplier or by requesting transshipments from other locations. We consider the problem to define heuristics for such ordering and transshipment decisions (OD and TD) that optimize given performance measures for the system.

The most common and broadly studied class of models assumes a single product, discrete review, independently and identically distributed demand, backlogging, complete pooling of product units between locations, emergency transshipments at the end of an order period, zero lead times, linear cost functions, and the total expected costs as performance measure (see **(author?)** [1]). In general analytical solutions are not feasible for MLIMTs due to the transshipments. TDs change the state of the system and thus influence the OD. Thus, the total consequences of an OD cannot be calculated directly. Approximate models and simulation are alternatives, see, e.g., [1–3]. Further Difficulties connected with TDs arise for continuous review models, e.g., the prevention of forth-and-back transshipments. Therefore, continuous MLIMTs are usually investigated under several simplifying assumptions, such as two locations [4, 5], Poisson demand [6], a fixed order policy not considering future transshipments [7], restriction to basic strategies such as a one-for-one ordering policy [6] and an all-or-nothing transshipment policy [4], or the constraint that at most one transshipment with insignificant time and a single shipping point is possible over an order period [5]. These approaches have not completely answered the question of optimal order and transshipment policies, because all models assume a certain order policy and heuristic transshipments. In a few cases simulation is employed to test approximate analytical models [5, 7] or to find the best reorder point s for a (s, S)-order policy [8] by linear search and simulation. Consequently, the results are restricted to models of small size.

The MLIMT presented here has been developed with respect to the connection of discrete review for ordering and continuous review for transshipments, creating a simulator as general as possible to abandon restrictions of existing studies and to ensure broad applicability, and iteratively connecting an MLIMT simulator with an evolutionary optimization algorithm to investigate the search space by simulation-based evolutionary optimization [9]. In these studies the system has been utilized to analyze a pairwise connected four-location model. Various criteria, compound renewal demand processes as well as arbitrary ordering, demand satisfaction and transshipment modes can be investigated (see [10, 11] for details).

II. Experimental Settings

Despite a simulation model can represent any real system with arbitrary accuracy, our objective is not to design a simulator for all multi-location inventory systems but to describe a simulator which is capable to find efficient solutions for the optimal control of an important class of MLIMTs in reasonable time. In addition, that class of MLIMTs goes beyond models which can be analytically investigated because optimal transshipment decisions are solutions of open (linear) transportation problems, which are not feasible analytically [10]. Hence, the application domain is extended considerably.

A. Number of Locations

With respect to the analytical tractability the cases $n = 2$ and $n > 2$ can be distinguished. The simulator is suited for MLIMTs with an arbitrary number of independent non-homogeneous locations, although the optimization task becomes more difficult for an increasing number. A longer simulation time is necessary to ensure sufficient accuracy, and a higher optimization cycle count is essential to converge to the global optimum. However, for a large-scale model some potential transshipments may be excluded prior to the

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optimization. Thus, potential transshipments can be limited to locations within certain pooling groups (cp. II-F Pooling Mode) to reduce the complexity of the optimization task.

B. Number of Products

There may be a single product or a finite number of different products, whereas a substitution order between products might be defined in the latter case. Most approaches as well as the MLIMT simulator assume a single product. For the consideration of multiple products, sequential simulation and optimization is feasible, unless fixed costs or finite resources are shared. However, this limitation is negligible provided that shared fixed costs are insignificant relative to total fixed and variable costs, and capacities for storage and transportation are considered to be infinite.

C. Ordering Mode

The ordering mode defines when to order, i.e., the *review scheme*, and what *order policy* to use. The review scheme defines the points in time for ordering, where discrete or continuous review is possible. Under the discrete review scheme the planning horizon is divided into periods, which is the case for most analytical models. We define a periodic review scheme with fixed length $t_{\mathrm{P},i}$ of the review period for orders and a continuous review scheme for transshipments at location i. Of course arbitrary order policies based on the inventory position at the end of the order period are possible. In the experimentation we focus on a (s_i, S_i)-order policy.

D. Demand Process

Demand may be deterministic or random, identical or different for all locations, stationary or non-stationary in time, independent of or dependent on locations and time as well as with complete or incomplete information. The most common class of models assumes demand independently and identically distributed over time. We define a *compound renewal demand process*, which is distinguished by two independent random variables: T_i for the inter-arrival time of clients at location i and B_i for their demand, $i = 1, 2, \ldots, n$. Thus, exact holding and penalty costs can be calculated, which is an extension of analytical models with discrete review, where the whole demand of a period is realized to the end of a period. That disadvantage does not exist for models with continuous review, but in almost all such models a Poisson demand process is assumed.

E. Demand Satisfaction Mode

The mode for demand satisfaction describes how to process arriving demand for each location. It is common to assume a queue for backlogged demand. In dependence on an infinite, finite, or zero queueing capacity there exist the backlogging, intermediate, and lost-sales cases. Waiting demand is served according to a service policy and may finally have a random impatient time. In the experimentation we concentrate on a FIFO (first in, first out) service policy for demonstration purposes. The impatient time W_i is a random variable arbitrarily distributed for each location i.

F. Pooling Mode

The pooling mode comprises all rules by which the on-hand stock is used to respond to shortages. Pooling may be complete or partial, defining which location and which quantity of product units are pooled. The MLIMT simulator includes all pooling modes from complete pooling to time-dependent partial pooling. A symmetric $n \times n$ matrix $\mathbf{P} = (p_{ii'})$ defines pooling groups in such a way that two locations i and i' belong to the same group iff $p_{ii'} = 1$ (otherwise $p_{ii'} = 0$). Transshipments are useful to balance shortages within an order cycle, but may be counterproductive near the end of such a cycle. Therefore, the pooling time $t_{\mathrm{pool},i} \in [0, t_{\mathrm{P},i}]$ is defined. After the k-th order request, location i may receive transshipments from all other locations as long as for the current time $t \leq k\, t_{\mathrm{P},i} + t_{\mathrm{pool},i}$ holds. Subsequently, location i may obtain product units only from locations within the same pooling group. Note that this parameter suppresses the effect of other transshipment parameters and thus increases the requirements on the evolutionary optimization. In the experiments we assume complete pooling.

G. Transshipment Mode

The objective of transshipments is to use product units efficiently for the whole system. Therefore, shortage and excess at different locations are balanced. There may be *preventive* lateral transshipments to anticipate a stock-out or *emergency* lateral transshipments. Most existing models define the latter ones at the end of a period. According to continuous review, the MLIMT simulator allows transshipments as well as multiple shipping points and partial deliveries. To control the flow of transshipments, a great variety of rules can be defined. Three basic ideas ensure broad applicability – *priorities*, generalizing common transshipment *rules* and introducing a *state function*. TDs should be based on appropriate forecasts for the dynamics of the model, especially the stock levels, to calculate their effects. For each location transshipment orders (TO) and product offers (PO) are distinguished. Moments for TOs or POs are the arrival events of clients or transshipments and order deliveries, respectively. *Priorities* define a sequence of transshipments in one-to-many and many-to-one situations. Because of continuous time only the following three *rules* occur: BAN (biggest amount next), MTC (minimum transshipment costs per unit), and MTT (minimum transshipment time). The priorities result then from combination of these rules. *State functions* are observed to decide if a TO or a PO should be released. The following notations for each location i and time $t \geq 0$ are useful for further statements: $y_i(t)$, inventory level; $y_i^{\pm}(t) = \max(\pm y_i(t), 0)$, on-hand stock $(+)$ and shortage $(-)$, respectively; $b_{\mathrm{ord},i}(t)$, product units ordered but not yet delivered; $b_{\mathrm{ord},k,i}$, product units ordered in the k-th request; $b_{\mathrm{tr},i}(t)$, transshipments on the way to location i; $r_i(t) = y_i(t) + b_{\mathrm{ord},i}(t) + b_{\mathrm{tr},i}(t)$, inventory position; $t_{\mathrm{P},i}$, order period time; $t_{\mathrm{A},i}$, delivery lead time of an order; $n_{ord,i} = \lfloor t_{\mathrm{A},i}/t_{\mathrm{P},i} \rfloor$, number of periods to deliver an order.

To decide at time t and location i about a TO or PO, the state functions $f_{\mathrm{TO},i}(t)$ and $f_{\mathrm{PO},i}(t)$ are defined based on the available stock plus expected transshipments $f_{\mathrm{TO},i}(t) = y_i(t) + b_{\mathrm{tr},i}(t)$ and the on-hand stock $f_{\mathrm{PO},i}(t) = y_i^+(t)$, respectively. A heuristic (h_i, H_i)-rule for TOs is proposed in the following way ($h_i \leq H_i$):

If $f_{\mathrm{TO},i}(t) < h_i$ then
release a TO for $H_i - f_{\mathrm{TO},i}(t)$ product units.

For positive transshipment times it might be advantageous to consider expected demand, i.e., a TO is released on the basis of the state function $f_i(t')$ for a time moment $t' \geq t$. The MLIMT simulator offers three such moments, the current time $t' = t$, the next order review time $t' = t_1$, and the next time of an order delivery $t' = t_2$. In the experimentation we focus on the comparison of time t and t_1. For example the state function $f_{\mathrm{TO},i}(t) = y_i(t) + b_{\mathrm{tr},i}(t)$, $t \geq 0$ is considered. Let $k\, t_{\mathrm{P},i} \leq t < (k+1)t_{\mathrm{P},i}$, i.e., the review period after the k-th order request. Then t_1 is defined as:

$$t_1 = (k+1)t_{\mathrm{P},i} \ . \tag{1}$$

For t_2 we introduce two events:

$ev(t) \leftrightarrow \{$in the actual period was no order delivery until $t\}$
$\overline{ev}(t) \leftrightarrow \{$there was an order delivery until $t\}$.

Then the following holds:

$$t_2 = (k - n_{\mathrm{ord},i})t_{\mathrm{P},i} + t_{\mathrm{A},i} + \begin{cases} 0 & \text{if } ev(t) \leftrightarrow t < (k - n_{\mathrm{ord},i})t_{\mathrm{P},i} + t_{\mathrm{A},i} \\ t_{\mathrm{P},i} & \text{if } \overline{ev}(t) \leftrightarrow t \geq (k - n_{\mathrm{ord},i})t_{\mathrm{P},i} + t_{\mathrm{A},i} \end{cases} . \tag{2}$$

Considering $m_i = \langle B_i \rangle / \langle T_i \rangle$, the expected demand per time unit at location i, the following forecasts are used:

$$\hat{f}_{\mathrm{TO},i}(t) = f_{\mathrm{TO},i}(t) = y_i(t) + b_{\mathrm{tr},i}(t) \ , \tag{3}$$

$$\hat{f}_{\mathrm{TO},i}(t_1) = f_{\mathrm{TO},i}(t) - m_i(t_1 - t) + \begin{cases} b_{\mathrm{ord},k',i}, \ k' = k - n_{\mathrm{ord},i} & \text{if } ev(t) \\ 0 & \text{if } \overline{ev}(t) \end{cases} , \tag{4}$$

$$\hat{f}_{\mathrm{TO},i}(t_2) = f_{\mathrm{TO},i}(t) - m_i(t_2 - t) \ . \tag{5}$$

Thus, replacing function $f_{\mathrm{TO},i}(t)$ by various forecast functions, a great variety of transshipment modes can be described. Note that in case of linear transshipment cost functions without set-up component, the (h_i, H_i)-rule degenerates to the (H_i, H_i)-rule, which is expected to be approximated by a well-designed evolutionary optimization algorithm.

To decide what quantity is offered at time t, the offering level o_i is introduced, corresponding to the hold-back level in **(author?)** [5]. Since only physically available stock is offered, the state function $f_{\mathrm{PO},i}(t) = y_i^+(t)$ is defined. To prevent undesirably small and frequent transshipments, the offered quantity $y_i^+(t) - o_i$ must not be smaller than $\Delta o_{\mathrm{min},i}$. Similar forecasts are applied with forecast times t, t_1 and t_2 (see **(author?)** [10] for details). Thus, the PO rule is as follows:

If $\hat{f}_{\mathrm{PO},i}(t) - o_i \geq \Delta o_{\mathrm{min},i}$ then
release a PO for $\hat{f}_{\mathrm{PO},i}(t) - o_i$ product units.

The most important generalizations compared to common approaches are: First, introducing various control parameters, we have extended the set of available transshipment policies considerably, including all common policies. Second, multiple shipping points with partial deliveries are allowed. Finally, introducing time-dependent forecasts, the proposed decision rules become non-stationary in time.

H. Lead Times

Lead times for order deliveries and transshipments of product units may be negligible, positive constants, or random. Most analytical models assume zero lead times, while the MLIMT simulator defines location-specific order lead times and a not necessarily symmetric distance matrix for all locations. In conjunction with the transport velocity, transshipment lead times are determined.

I. Cost and Gain Functions

There may occur costs for ordering, storing, and transshipping product units as well as for waiting and lost demand. These functions may be linear, linear with set-up part, or generally non-linear. A location may also receive gain from product units sold to clients. To solve models analytically, often linear cost functions are assumed. With respect to the MLIMT simulator, order cost functions, holding cost functions, shortage cost functions and transshipment cost functions contain components which are fixed as well as linear in time and quantity. Fixed costs arise from each non-served demand unit. All cost parameters are location-specific. The gain from a unit sold to a client is a constant.

J. Planning Horizon

The planning horizon may be finite or infinite. In case of periodic review it may consist of a single period. Simulating infinite planning horizons by a finite simulation is of course not possible. Thus, appropriate approximations are used. An adequate estimation of the stationary properties can be achieved by a sufficiently long simulation time. The only difficulty is the fast increase of necessary computation time.

K. Objective Function

As objective function, various cost criteria can be formulated, such as total expected costs, total expected discounted costs, long-run average costs, and non-cost criteria like service rates or expected waiting times. Both criteria types may form a multi-objective problem. Alternatively, one criterion can be optimized while given restrictions have to be satisfied. Existing models commonly observe the total expected costs criterion. For the MLIMT simulator different objectives can be optimized by choosing corresponding cost functions. For example the service rate or average waiting times can be expressed by specific shortage cost functions. Thus, several criteria are applicable.

III. Experimental Results

To get an impression of simulation-based evolutionary optimization applied to multi-location inventory models with lateral transshipments, results of the Genetic Algorithm (see Algorithm 1) for two experiments on a four-location model are discussed in this section. The system topology is visualized in Fig. 1. In both experiments we assume identical location-dependent model characteristics, excluding the client demand distributions, which are uniform distributions in example 1 and exponential distributions in example 2. However, equal expected values are assumed. Although the simulation-based evolutionary optimization approach is applicable to in fact very general models, the following assumptions are considered due to limited space:

1) All locations i use a (s_i, S_i)-order policy. For all locations i an order period is equal to 10 days, i.e., $t_{\mathrm{P},i} = 10$ days. The order lead times are $t_{\mathrm{A},1} = 48\,\mathrm{h}$, $t_{\mathrm{A},2} = 60\,\mathrm{h}$, $t_{\mathrm{A},3} = 72\,\mathrm{h}$, and $t_{\mathrm{A},4} = 84\,\mathrm{h}$ for locations 1 to 4, respectively.
2) Transshipment times are $8.66\,\mathrm{h}$ between the outer locations 1–3 and $5\,\mathrm{h}$ between an outer location and the central location 4. The priority sequence for transshipment orders and product offers is MTC, BAN, MTT. Stocks of all locations i are completely pooled, i.e., $t_{\mathrm{pool},i} = t_{\mathrm{P},i}$, and transshipments are therefore not constrained to certain pooling groups.
3) The inter-arrival time of customers to each location i is an exponentially distributed random variable with $\langle T_i \rangle = 2\,\mathrm{h}$. The customer demand D_i is uniformly distributed in the interval $(0, 5(i+1))$ in example 1 and exponentially distributed with mean $\langle D_i \rangle = 2.5(i+1)$ in example 2 for each location i. The service policy is FIFO, and the impatient time is triangularly distributed in the interval $(0\,\mathrm{h}, 8\,\mathrm{h})$, thus, $\langle W_i \rangle = 4\,\mathrm{h}$.
4) The inventory costs are 1€ per unit and day, whereas the linear order and transshipment costs are 1€ per unit and per day of transportation time. The fixed transshipment costs equal 500€ for each location and the gain per unit sold is 100€. Out-of-stock costs are fixed to 50€ per lost client and 1€ per hour. The objective function is total costs over the simulation time of 260 weeks excluding the transition time of 52 weeks.

The state function chosen for transshipment orders and product offers is $f_{\mathrm{TO},i}(t) = y_i(t) + b_{\mathrm{tr},i}(t)$ and $f_{\mathrm{PO},i}(t) = y_i^+(t)$, respectively. To analyze the effect of forecasting demand, all four combinations of the current time t and the forecast moment t_1 are compared for both state functions. For optimization Algorithm 1 is used with a population of 50 individuals, where an individual is a candidate solution, i.e., a vector of all policy parameters. The optimization stops if a new optimum has not occurred for the last 1,000 cycles. However, at least 3,000 cycles are realized to prevent early convergence to a local optimum and to find the global optimum, though good values are achieved fast.

For the initialization of the Genetic Algorithm inventory related values are set in accordance with the capacity $y^+_{\max,i}$ for each location i. The reorder level s_i is uniformly distributed in the interval $[-y^+_{\max,i}, y^+_{\max,i})$. In case of a (s_i, S_i)-order policy, the order-up-to level S_i is uniformly distributed in the interval $(s_i, s_i + 2y^+_{\max,i})$. Similarly the (h_i, H_i)-transshipment request policy is initialized for each location i. Thus, the request level h_i and the request-up-to level H_i are uniformly distributed in the intervals $[-y^+_{\max,i}, y^+_{\max,i})$ and $(h_i, h_i + 2y^+_{\max,i})$, respectively. The product offer policy defines an offer level o_i, uniformly distributed in the interval $[-y^+_{\max,i}, y^+_{\max,i})$, and a minimum offer quantity $\Delta o_{\min,i}$, uniformly distributed in the interval $(0, y^+_{\max,i}/4)$.

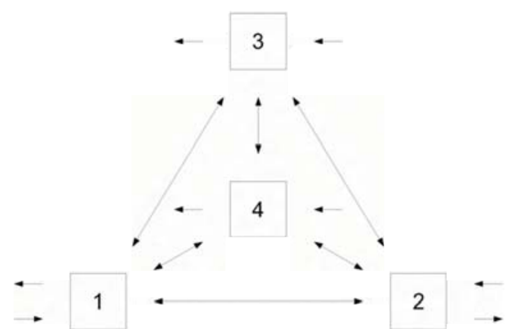

Fig. 1. System Topology.

For all experiments the overall results, optimized parameter values and cost function values are discussed. Prohibitive parameter values are enclosed in [brackets], i.e., values that prevent ordering or offering product units. In general the Genetic Algorithm may choose values arbitrarily if there is no difference in the objective function. This is the case for values below or above a certain limit. Thus, after the optimization, the minimum absolute values of all parameters not changing the cost function values are determined using binary search.

Algorithm 1 Elitist Genetic Algorithm

generate an initial population $\boldsymbol{\alpha}$
calculate selection quantity m according to recombination probability P_C
for $t \leftarrow 1$ **to** ∞ **do**
    $G, H, L \leftarrow \emptyset$
    $G \leftarrow$ choose the best of all M current individuals $\boldsymbol{\alpha}$
    $H \leftarrow$ choose by stochastic universal sampling $m - 1$ individuals in $\boldsymbol{\alpha}$
    $L \leftarrow$ generate $M - m$ individuals by recombining uniformly from H
    for all individuals from H and L **do**
        select the individuals to mutate with probability P_M
        change each value x of selected ones with probability $P_{M,x}$
        add the individual to G
    individuals in G define the new generation $\boldsymbol{\alpha}$ of size M
    if $(terminating\_condition)$ **then**
        return $\boldsymbol{\alpha}$

A. Example 1 – Uniformly Distributed Demand

Table I shows the parameter setup for example 1 and Table II the equivalent results. For uniformly distributed demand, solution 3 gives the least total costs, benefiting from demand forecast for transshipment order decisions. This solution also shows the least out-of-stock costs, followed by solution 2. Except for solution 2, all solutions use transshipments to compensate shortages. However, only solution 4 shows a directed flow of transshipments from location 4 to location 2, whereas transshipments in both directions are observed in case of solutions 1 and 2. Thus, the system has evolved an implicit structure, though all locations share identical parameters at the beginning. For all solutions displaying transshipments there is one location controlling its inventory level completely via transshipments instead of releasing periodic orders.

B. Example 2 – Exponentially Distributed Demand

Table III shows the parameter setup for example 2 and Table IV the equivalent results. For exponentially distributed demand, solutions 2 and 3 show the lowest total costs and lowest out-of-stock costs, while the first one uses demand forecast for product offers and thus leads to less out-of-stock costs. The latter one applies demand forecast for transshipment orders resulting in less total costs. For all solutions transshipments are observed, but only solution 2 shows a directed flow. In case of the solutions 1, 3 and 4, alternating transshipments are released, i.e., there are flows in both directions. Again, the development of an implicit structure can be observed. In each case there is one location substituting periodic orders completely by transshipments from other locations.

IV. Conclusions and Future Work

It has been demonstrated that a four-location model with lateral transshipments can be optimized applying simulation-based evolutionary optimization, and the practitioner gets a detailed description how to achieve this. Further, we have shown that the demand distribution has a significant impact on the optimization result. Thus, investigations of more complex and empirically derived distribution functions are expected to lead to parameters that are suited best for certain classes of models. A major benefit of applying simulation-based optimization is that simulation models and the layout of the system under investigation can be changed or extended relatively straightforward.

For the future it would be interesting to standardize the approach and to offer services for the simulation-based optimization of given systems. Functional extensions of the model include new policies for periodic orders, transshipment orders, and product offers. Extensions of the parameter set itself comprise the capacity of the locations. Therefore, the introduction of costs for unused storage is necessary, constituting estate and energy costs. Thus, the analysis of such a system results not only in optimal parameters to control the flows of product units, but also in a reallocation of capacities. In addition to these static aspects of the model, dynamic properties such as the location-specific order period time might be added to the parameter set. However, there certainly are restrictions in real-world applications.

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TABLE I

PARAMETER VALUES AND COST FUNCTION VALUES FOR EXAMPLE 1.

		Periodic order		Transship. order		Product offer		Inventory	Out-of-stock	Periodic order	Transshipment	
	i	s_i	S_i	h_i	H_i	o_i	$\Delta o_{\min,i}$	costs in €	costs in €	costs in €	costs in €	Gain in €
1	1	356	795	[0]	[0]	[0]	[806]	557,741	7,155	244,099	0	8,613,278
	2	[0]	[0]	914	1,462	0	1,042	816,789	9,723	0	25,543	13,062,067
	3	1,200	1,701	1,250	1,250	[297]	[1,216]	1,136,491	4,016	573,325	0	17,483,570
	4	1,019	3,124	1,573	2,043	1,577	0	1,314,275	15,885	1,311,993	51,619	21,794,912
	$\sum$							3,825,296	36,779	2,129,417	77,162	60,953,827
2	1	365	804	[0]	[0]	[0]	[317]	570,168	5,684	244,321	0	8,624,377
	2	578	1,257	[0]	[0]	[0]	[436]	858,426	6,371	398,649	0	13,087,921
	3	887	1,732	[0]	[0]	[0]	[623]	1,124,607	9,541	593,731	0	17,422,405
	4	1,093	2,244	[0]	[0]	[0]	[1,332]	1,413,377	9,347	833,417	0	21,871,808
	$\sum$							3,966,578	30,942	2,070,118	0	61,006,512
3	1	379	818	[–48]	[0]	[135]	[663]	588,592	3,762	244,560	0	8,636,330
	2	576	1,255	[0]	[0]	[0]	[1,241]	855,411	6,647	398,600	0	13,085,942
	3	1,200	3,224	0	1,276	208	1,057	891,662	6,703	1,246,027	169,650	17,460,357
	4	[0]	[0]	948	2,439	1,526	0	1,312,262	12,459	0	124,626	21,827,396
	$\sum$							3,647,927	29,570	1,889,186	294,276	61,010,025
4	1	367	806	[0]	[0]	[0]	[355]	572,078	5,272	244,349	0	8,625,792
	2	[0]	[0]	312	1,205	[0]	[0]	898,369	50,474	0	0	12,702,304
	3	885	1,730	[0]	[0]	[472]	[427]	1,121,349	9,763	593,651	0	17,419,721
	4	1,116	3,191	[0]	[0]	815	321	1,512,392	1,339	1,279,337	27,794	21,954,773
	$\sum$							4,104,188	66,848	2,117,337	27,794	60,702,591

TABLE II

TOTAL RESULTS FOR EXAMPLE 1.

	Transshipment order	Product offer	Result	Cycle	Rank
1	current time t	current time t	–54,885,174	1,784 (3,000)	3
2	current time t	time of next order t_1	–54,938,874	1,784 (3,000)	2
3	time of next order t_1	current time t	–55,149,067	1,784 (3,000)	1
4	time of next order t_1	time of next order t_1	–54,386,423	2,710 (3,710)	4

TABLE III

PARAMETER VALUES AND COST FUNCTION VALUES FOR EXAMPLE 2.

		Periodic order		Transship. order		Product offer		Inventory	Out-of-stock	Periodic order	Transshipment	
	i	s_i	S_i	h_i	H_i	o_i	$\Delta o_{\min,i}$	costs in €	costs in €	costs in €	costs in €	Gain in €
1	1	428	832	[0]	[0]	[0]	[823]	611,801	4,805	243,755	0	8,589,685
	2	664	1,288	[0]	[0]	[0]	[1,286]	900,869	7,816	399,531	0	13,123,242
	3	[0]	[0]	826	2,237	0	1,386	1,193,483	27,144	0	8,864	17,159,198
	4	1,369	3,468	1,269	3,259	1,685	0	1,452,823	9,624	1,429,314	45,857	21,762,930
	$\sum$							4,158,977	49,389	2,072,600	54,721	60,635,056
2	1	[0]	[0]	610	642	[0]	[523]	604,484	2,512	0	0	8,601,402
	2	654	1,278	[0]	[0]	[0]	[656]	886,560	8,859	399,252	0	13,112,049
	3	868	1,700	1,157	1,187	[0]	[889]	1,211,481	1,644	545,235	0	17,422,146
	4	1,131	3,003	[0]	[0]	–839	1,537	1,539,422	6,566	1,187,311	30,139	21,793,064
	$\sum$							4,241,946	19,581	2,131,797	30,139	60,928,660
3	1	368	1,019	[0]	[0]	703	0	591,947	6,227	301,900	7,522	8,586,351
	2	632	1,256	[0]	[0]	[0]	[1,254]	858,075	11,975	398,600	0	13,085,992
	3	1,200	3,161	0	1,684	120	1,164	870,787	2,546	1,158,381	300,270	17,410,444
	4	[0]	[0]	991	2,773	1,386	0	1,331,795	2,571	0	261,276	21,841,466
	$\sum$							3,652,603	23,320	1,858,882	569,068	60,924,253
4	1	421	825	[0]	[0]	[0]	[423]	602,220	5,622	243,645	0	8,584,211
	2	1,021	2,988	946	1,010	–675	1,432	1,057,626	49,876	951,486	86,419	12,735,925
	3	1,433	2,819	1,643	1,670	[1,082]	[931]	2,009,390	6,876	531,654	0	17,375,827
	4	[0]	[0]	0	2,153	643	68	1,355,478	15,394	0	41,249	21,698,890
	$\sum$							5,024,714	77,768	1,726,784	127,667	60,394,853

TABLE IV

TOTAL RESULTS FOR EXAMPLE 2.

	Transshipment order	Product offer	Result	Cycle	Rank
1	current time t	current time t	–54,299,368	1,784 (3,000)	3
2	current time t	time of next order t_1	–54,505,197	1,784 (3,000)	2
3	time of next order t_1	current time t	–54,820,380	1,592 (3,000)	1
4	time of next order t_1	time of next order t_1	–53,437,920	1,784 (3,000)	4