On Spatial Multiplexing of Strong Line-of-Sight MIMO With 3D Antenna Arrangements

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***Abstract*—In recent works, it has been shown that specific 2D antenna arrangements for multiple-input multiple-output (MIMO) systems can achieve similarly high spatial multiplexing gains under *deterministic* line-of-sight (LOS) conditions as non-line-of-sight channels with strong scattering considered in classical papers. However, the question whether 3D antenna arrays could provide an additional advantage was not addressed. In this work we show that the capacity of dominant LOS MIMO channels is invariant w.r.t. small offsets of the antenna elements along the transmit direction. This proves that the optimal 2D arrangements for point-to-point communication of LOS MIMO arrays are equivalent to 3D arrangements, whose projections of the antenna positions into a plane perpendicular to the transmit direction reproduce the optimal 2D arrangements. This insight also leads directly to the optimal designs for antenna arrays that communicate with each other along a transmit direction that is *oblique* w.r.t. the array plane(s).**

***Index Terms*—Strong LOS channel, MIMO, 3D antenna arrangements, 3D topology, capacity invariant, optimal antenna arrangements, curved surfaces and arbitrary tilted planes.**

I. Introduction

The unlicensed large bandwidth at frequencies around 60 GHz is an interesting solution to overcome data rate bottlenecks of future wireless communications systems. Especially 60 GHz is quite suitable for small cell backhaul systems due to the high interference immunity and security from strong path loss even at high frequency reuse. In addition, MIMO transmission has become an essential technique to provide the high data rates demanded by users of modern wireless communications systems. Last but not least: due to the high frequency, millimeter-wave wireless communication systems also have reasonable antenna sizes that may help to exploit spatial multiplexing gains in deterministic LOS channels, beyond the power gains achievable with beamforming.

As an example, for the channel that occurs in such a situation, we cite Ref. [1] (Section III). It was shown that at a frequency of 2 GHz (wavelength $\lambda \simeq 15$ cm), the antenna spacing of transceivers for a 3×3 MIMO system is optimized at 1 m for a transmit distance of $D = 20$ m. A similar calculation for 60 GHz ($\lambda \simeq 0.5$ cm) shows that the transmit distance should/could be extended to 600 m for the same 3×3 MIMO system with the antenna spacing of 1 m. This implies that LOS MIMO point-to-point communication at millimeter waves is promising for wireless backhaul systems.

Most of the literature on MIMO communication systems following the initial work of *Telatar* [2] focuses on channel scenarios with rich scattering in non-line-of-sight situations that are modeled stochastically.[1] Due to the more or less stationary situation in fixed wireless access applications and high gain antennas with high directivity, we will use a *deterministic* MIMO channel model with sub-channels using the spherical-wave model in the following that only takes geometrical properties of the antenna arrays at transmitter and receiver side into account. A recent published work [3] showed that spherical-wave model are more accurate and optimistic than the conventional approach using plane-wave model.

Other previous studies [4]–[7] showed, as one might expect, that *deterministic* LOS MIMO systems without specific geometrical arrangements result in channel matrices $\mathbf{H}$ with undesirable low singular values (in contrast to equal magnitude and associated small condition number of $\mathbf{HH}^H$). Those channel matrices with additional interference coupling between the data streams, due to unavoidable deviations from exact orthogonality, are therefore incapable of exploiting the potential spatial multiplexing gain. To avoid such channel matrices, in the same studies [4]–[6] it was shown that special 2D antenna arrangements work quite well in LOS environments. The works in [5], [6] have indicated that optimal LOS MIMO antenna arrangements in 1D or 2D mainly depend on operating frequency, number of antenna elements and transmit distance. For the parallel linear arrays, the optimal antenna spacing of uniform linear arrays (ULA) d_t and d_r at transmitter and receiver side given by [1], [8] satisfy

$$d_\mathrm{t} \cdot d_\mathrm{r} = \frac{\lambda D}{M}, \tag{1}$$

where the ULA consists of M antennas. D denotes the desired range of link. λ is the wavelength of the carrier. Meanwhile, the optimal antenna spacing $d_\mathrm{t,v}$, $d_\mathrm{r,v}$, $d_\mathrm{t,h}$, and $d_\mathrm{r,h}$ of uniform rectangular arrays (URA) in horizontal and vertical directions of the parallel transmitter and receiver arrays given by [5] satisfy

$$d_\mathrm{t,v} \cdot d_\mathrm{r,v} = \frac{\lambda D}{M_\mathrm{V}}, \;\; d_\mathrm{t,h} \cdot d_\mathrm{r,h} = \frac{\lambda D}{M_\mathrm{H}}, \tag{2}$$

where the URA consists of $M_\mathrm{V} \times M_\mathrm{H}$ antennas.

Recent published work [3], [9] showed that for linear array, the large array orientations in 3D space have a significant impact on the capacity using a 2×2 deterministic MIMO channels as an example. Studies of the influence of offsets in transmit direction (such that the antennas become arranged in 3D) on

[1]Independent and identically distributed (i.i.d.) frequency flat Rayleigh channels leading to maximum capacity can be considered as a reference case, although this would only be rarely realized in practice.

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optimal LOS MIMO systems design have not led to conclusive results. The benefits of placing the antennas on a 3D polyhedron under LOS indoor conditions with reflection components was presented in [10], showing that the dependency of capacity upon the rotation angles is significantly reduced. However, the optimality of the setups was not demonstrated. In [11], the information capacity per antenna for ULA, URA and a uniform cubic array (UCA) w.r.t. the number of antennas was presented. It was shown that, for large numbers of antennas with ULA and URA arrangement, the capacities per antenna becomes constant, while for a UCA the capacity per antenna decreased exponentially w.r.t the number of antennas. However, only an intuitive interpretation was given by suggesting that only the surface layer of 3D MIMO arrays contributes to the capacity. This phenomenon is explained by our work in a later section.

The sensitivity of the deterministic MIMO channels and comparisons with statistic MIMO channels have been well studied by Bøhagen *et al.* [4], [12], [13] as well as by Larsson [6]. It has shown that the capacity of a deterministic LOS MIMO channel under optimal antenna arrangement is only weakly sensitive to deviations induced by small tilts or translations. Finally, for obliquely oriented uniform planar arrays (UPA) optimal designs in terms of URA were shown in [13] for certain specific array orientations. We generalize these results by showing that the second principal direction of the UPA is not necessarily orthogonal to the first one, because the projection of the optimal orthogonal design onto a tilted plane leads to parallelograms. This means that the optimum antenna arrangement for 2D MIMO system is not generally a URA.

II. Channel Model

We consider a deterministic LOS MIMO channel with a total of N transmit antennas $(\mathrm{T_x})$ and a total of M receive antennas $(\mathrm{R_x})$. For a both time-flat and frequency-flat MIMO channel, the received signal can be described as

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{3}$$

where $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is the transmitted signal, ρ is the common power attenuation[2] $\mathbf{n}$ is the white complex Gaussian noise vector with $\mathbf{n} \sim \mathcal{CN}^{M\times 1}(0, \sigma_{\mathrm{n}}^2)$, $\mathbf{H} \in \mathbb{C}^{M\times N}$ is the normalized channel matrix, which implies that each element h_{mn} in $\mathbf{H}$ has unit channel gain, and the average signal-to-noise ratio (SNR) is independent of $\mathbf{H}$. Therefore, the normalized channel coefficient between equally polarized transmit antenna m and receive antenna n is modeled as

$$h_{mn} \triangleq e^{-j\frac{2\pi}{\lambda} r_{mn}}, \tag{4}$$

where the r_{mn} is the distance between the two antennas.

Assuming that the channel state information is only available at $\mathrm{R_x}$, the channel capacity of the MIMO transmission given by [2], [14] is formulated as

$$C = \log_2 \det\left(\mathbf{I}_M + \frac{\bar{\gamma}}{N}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right), \tag{5}$$

where $(\boldsymbol{.})^{\mathrm{H}}$ is the Hermitian transpose operator $\bar{\gamma}$ is the average total SNR at each receive antenna.

[2]The differences between the attenuation factors over sub-channels are omitted due to a reasonable assumption that the intra-array distances are much smaller than the transmit distance [5].

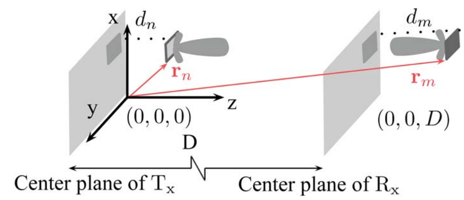

Fig. 1. System model.

III. 3D Antenna Arrangement and Channel Matrix

In this section, the influence of shifting antennas in the transmit direction is investigated. The system under investigation consists of two opposing antenna arrays separated by distance D. For simplicity, we consider one high directivity antenna n of transmit antenna array $(\mathrm{T_x})$ and one high directivity antenna m of receive antenna array $(\mathrm{R_x})$, as indicated in Fig. 1. Additionally, assume that the phase center of the transmitter antenna array is located at (0, 0, 0) and the transmit direction is along the z-axis. Therefore, the center of the receive antenna is located at $(0, 0, D)$. The position of the antenna m and n can be described by the vectors

$$\mathbf{r}_n = (x_n, y_n, d_n), \;\; \mathbf{r}_m = (x_m, y_m, D + d_m), \tag{6}$$

where we assume that the additional displacement d_m, d_n and the antenna aperture, the area of the projection of the array in the xy-plane, are much smaller than the transmit distance, $x_m, y_m, d_m, x_n, y_n, d_n \ll D$. The antenna distance r_{mn} between antenna m and n, which determines the entries of the channel matrix and ultimately the link performance according to Equation (4), may be written as

$$\begin{aligned} r_{mn} &= |\mathbf{r}_m - \mathbf{r}_n|_2 \\ &= \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2 + (D + d_m - d_n)^2} \\ &= (D+d_m-d_n)\cdot\left[1+\left(\frac{x_m-x_n}{D+d_m-d_n}\right)^2+\left(\frac{y_m-y_n}{D+d_m-d_n}\right)^2\right]^{\frac{1}{2}}. \end{aligned} \tag{7}$$

Applying a first order Taylor expansion to the square root and considering $x_m, y_m, d_m, x_n, y_n, d_n \ll D$, the antenna distance r_{mn} becomes

$$\begin{aligned} r_{mn} &\approx D+d_m-d_n+\frac{(x_m-x_n)^2}{2(D+d_m-d_n)}+\frac{(y_m-y_n)^2}{2(D+d_m-d_n)} \\ &\approx D+d_m-d_n+\frac{(x_m-x_n)^2}{2D}+\frac{(y_m-y_n)^2}{2D}. \end{aligned} \tag{8}$$

Without loss of generality, the phase shift caused by D can be dropped as it is a constant and the same for all paths. One may write Equation (4) as

$$h_{mn} \propto e^{-j\frac{2\pi}{\lambda}(d_m-d_n)} e^{-j\frac{2\pi}{\lambda}\frac{(x_m-x_n)^2+(y_m-y_n)^2}{2D}}. \tag{9}$$

The channel gain consists of two independent parts, the linear channel gain $h_{mn}^{\|}$ caused by the offsets in the transmit direction and the planar channel gain $h_{mn}^{\perp}$ caused by the antenna spacing on the plane that is perpendicular to the transmit direction (broadside relative to the transmit direction). The two parts follow as

$$h_{mn}^{\|} \triangleq e^{-j\frac{2\pi}{\lambda}(d_m-d_n)}, \tag{10}$$

$$h_{mn}^{\perp} \triangleq e^{-j\frac{2\pi}{\lambda}\frac{(x_m-x_n)^2+(y_m-y_n)^2}{2D}}, \tag{11}$$

which form the complete channel gain through

$$h_{mn} \propto h^{\parallel}_{mn} \cdot h^{\perp}_{mn}. \tag{12}$$

Consequently, when there are offsets along the z-direction, the channel matrix $\mathbf{H}$ can directly be decomposed into

$$\mathbf{H} = \mathbf{D}_{\parallel,\mathrm{r}} \cdot \mathbf{H}_{\perp} \cdot \mathbf{D}_{\parallel,\mathrm{t}}, \tag{13}$$

where $\mathbf{D}_{\parallel,\mathrm{t}}$, $\mathbf{D}_{\parallel,\mathrm{r}}$ are diagonal matrices that represent the phase shift caused by the offsets along the transmit direction at $\mathrm{T_x}$ and $\mathrm{R_x}$ respectively, $(\mathbf{D}_{\parallel,\mathrm{t}})_{nn} \triangleq e^{j\frac{2\pi d_n}{\lambda}}$, $n \in [1, N]$, $(\mathbf{D}_{\parallel,\mathrm{r}})_{mm} \triangleq e^{-j\frac{2\pi d_m}{\lambda}}$, $m \in [1, M]$ and $\mathbf{H}_{\perp}$ is the channel matrix contributed by the spatial multiplexing on the broadside relative to the transmit direction with $(\mathbf{H}_{\perp})_{mn} \triangleq h^{\perp}_{mn}$.

In this case, the channel capacity, Equation (5), is given by

$$\begin{aligned} C &= \log_2 \det\left(\mathbf{I}_M + \frac{\bar{\gamma}}{N}\mathbf{D}_{\parallel,\mathrm{r}} \cdot \mathbf{H}_{\perp} \cdot \mathbf{D}_{\parallel,\mathrm{t}} \cdot \mathbf{D}^{\mathrm{H}}_{\parallel,\mathrm{t}} \cdot \mathbf{H}^{\mathrm{H}}_{\perp} \cdot \mathbf{D}^{\mathrm{H}}_{\parallel,\mathrm{r}}\right) \\ &= \log_2 \det\left(\mathbf{I}_M + \frac{\bar{\gamma}}{N}\mathbf{D}_{\parallel,\mathrm{r}} \cdot \mathbf{H}_{\perp} \cdot \mathbf{H}^{\mathrm{H}}_{\perp} \cdot \mathbf{D}^{\mathrm{H}}_{\parallel,\mathrm{r}}\right) \\ &= \log_2 \det\left(\mathbf{I}_M + \frac{\bar{\gamma}}{N}\mathbf{H}_{\perp} \cdot \mathbf{H}^{\mathrm{H}}_{\perp} \cdot \mathbf{D}^{\mathrm{H}}_{\parallel,\mathrm{r}} \cdot \mathbf{D}_{\parallel,\mathrm{r}}\right) \\ &= \log_2 \det\left(\mathbf{I}_M + \frac{\bar{\gamma}}{N}\mathbf{H}_{\perp} \cdot \mathbf{H}^{\mathrm{H}}_{\perp}\right), \end{aligned} \tag{14}$$

where the third equivalent follows from determinant identity $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$.

Furthermore, considering singular values of the channel matrix $\mathbf{H}$, the singular value matrix $\mathbf{\Sigma}$ is equivalent to singular value matrix $\mathbf{\Sigma}_{\perp}$ of $\mathbf{H}_{\perp}$ as

$$\begin{aligned} \mathbf{H} &= \mathbf{D}_{\parallel,\mathrm{r}} \cdot \mathbf{H}_{\perp} \cdot \mathbf{D}_{\parallel,\mathrm{t}} \\ &= \mathbf{D}_{\parallel,\mathrm{r}} \cdot \mathbf{U}_{\perp}\mathbf{\Sigma}_{\perp}\mathbf{V}^{\mathrm{H}}_{\perp} \cdot \mathbf{D}_{\parallel,\mathrm{t}} \\ &= \mathbf{U}\mathbf{\Sigma}_{\perp}\mathbf{V}^{\mathrm{H}} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}, \end{aligned} \tag{15}$$

where $\mathbf{H}_{\perp}$ has singular value decomposition $\mathbf{H}_{\perp} = \mathbf{U}_{\perp}\mathbf{\Sigma}_{\perp}\mathbf{V}^{\mathrm{H}}_{\perp}$. $\mathbf{U} \triangleq \mathbf{D}_{\parallel,\mathrm{r}} \cdot \mathbf{U}_{\perp}$ and $\mathbf{V} \triangleq \mathbf{D}^{\mathrm{H}}_{\parallel,\mathrm{t}} \cdot \mathbf{V}_{\perp}$ are still unitary matrices as $\mathbf{U}\mathbf{U}^{\mathrm{H}} = \mathbf{U}^{\mathrm{H}}\mathbf{U} = \mathbf{I}$ and $\mathbf{V}\mathbf{V}^{\mathrm{H}} = \mathbf{V}^{\mathrm{H}}\mathbf{V} = \mathbf{I}$.

According to (14) and (15), the capacity of the MIMO channel is invariant with respect to the offsets in the transmit direction, as the singular values of $\mathbf{H}$ are only affected by $\mathbf{H}_{\perp}$ under the approximation in Equation (8). For any MIMO array with 3D antenna arrangement or 1D or 2D MIMO array with out-of-plane rotation, the $\mathbf{D}_{\parallel,\mathrm{t}}$, $\mathbf{D}_{\parallel,\mathrm{r}}$ terms introduced by the offsets can be eliminated through fixed pre-coder and equalizer with compute complexity of $O(N)$ and $O(M)$ easily.

From Equation (14), it follows that the offsets are not affecting the capacity contribution from broadside relative to the transmit direction. Therefore, the optimized solution for point-to-point 2D LOS MIMO array given by [5], [6] is also the best solution for point-to-point 3D LOS MIMO array if one wishes to achieve the highest capacity with a given antenna number. However, if one considers the projection of the antenna arrangements into multi-directions, 3D antenna structures are promising to provide better solutions in network based MIMO system with point-to-multipoints LOS communication, e.g. multi-access wireless backhaul, and point-to-point communication with reflection components in scattering environments that was also suggested by works in [10].

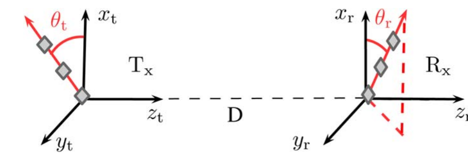

Fig. 2. Oblique ULA. Two parallel Cartesian coordinate systems are located at $\mathrm{T_x}$ and $\mathrm{R_x}$ with origins at the lower ends of the ULA. The z-axes are both in direction from $\mathrm{T_x}$ to $\mathrm{R_x}$. The $\mathrm{T_x}$ array is located in the xz-plane.

IV. Optimal Spatial Multiplexing Gain for Oblique Arrays

It is obvious from Equation (14) and (15) that the capacity of any 1D or 2D MIMO oblique arrays can be easily obtained via projecting onto broadside relative to the transmit direction. Thus, the optimal solutions of the antenna spacing under different out-of-plane rotation can also be obtained via mapping the broadside solution for transmit direction back to the plane where the arrays are located.

A. Uniform Linear Array

In this part, the antenna spacings d_t and d_r at the transmitter and receiver side separately are considered. The virtual arrays at the $\mathrm{T_x}$ and $\mathrm{R_x}$ have elevation angles θ_t and θ_r in their local spherical coordinate systems, as indicated in Fig. 2. The solution considers any arbitrary rotation of the ULA was given by the work [1], [8]. For the sake of completeness, the solution will be derived as well using the method mentioned above. Considering Equation (1) and mapping the arrangement of the parallel solutions back to the array, the antenna spacing satisfies the relation

$$d_\mathrm{t} \cdot d_\mathrm{r} = \frac{\lambda D}{\cos\theta_\mathrm{t}\cos\theta_\mathrm{r} M}. \tag{16}$$

B. Uniform Planar Array

In this part, the optimal UPA antenna arrangement under any rotation angles that the plane does not contain the transmission direction is investigated to compensate the degradation from tilt. Considering a 2D MIMO system as illustrated in Fig. 3, two parallel Cartesian coordinate systems $(x_\mathrm{t}, y_\mathrm{t}, z_\mathrm{t})$ and $(x_\mathrm{r}, y_\mathrm{r}, z_\mathrm{r})$ are constructed at $\mathrm{T_x}$ and $\mathrm{R_x}$. The z-axes are both in direction from $\mathrm{T_x}$ to $\mathrm{R_x}$. The y-axes can be chosen freely, for instance, parallel to the ground. In the oblique MIMO systems, transceivers are intended to be placed on the targeting plane $\mathrm{P}(n^i_\mathrm{x}, n^i_\mathrm{y}, n^i_\mathrm{z})$, $n^i_\mathrm{z} \neq 0$ and achieving the optimal spatial multiplexing gain. The $(n^i_\mathrm{x}, n^i_\mathrm{y}, n^i_\mathrm{z})^T$ indicates the normal vector $\mathbf{n}_{x'_i y'_i}$ of the plane in Cartesian coordinate system (x_i, y_i, z_i), $i \in \{\mathrm{t}, \mathrm{r}\}$. In the rest of this part, the coordinates (x'_i, y'_i) of antenna arrangement on the targeting plane is derived.

Assuming that the transformation from coordinate systems (x'_i, y'_i, z'_i) to (x_i, y_i, z_i) is realized via rotation matrix

$$\begin{aligned} \mathbf{Q}_i &= \mathbf{Q}_{\mathbf{y}'_\mathrm{i}}\mathbf{Q}_{\mathbf{x}'_\mathrm{i}} \\ &= \begin{pmatrix} \cos\varphi_i & 0 & \sin\varphi_i \\ 0 & 1 & 0 \\ -\sin\varphi_i & 0 & \cos\varphi_i \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta_i & \sin\vartheta_i \\ 0 & -\sin\vartheta_i & \cos\vartheta_i \end{pmatrix}, \end{aligned} \tag{17}$$

where ϑ_i, φ_i are the rotate angles about the x'_i-axis, y'_i-axis using right hand rule. Due to relation between the two Cartesian coordinate system (x_i, y_i, z_i) and (x'_i, y'_i, z'_i), the normal vector

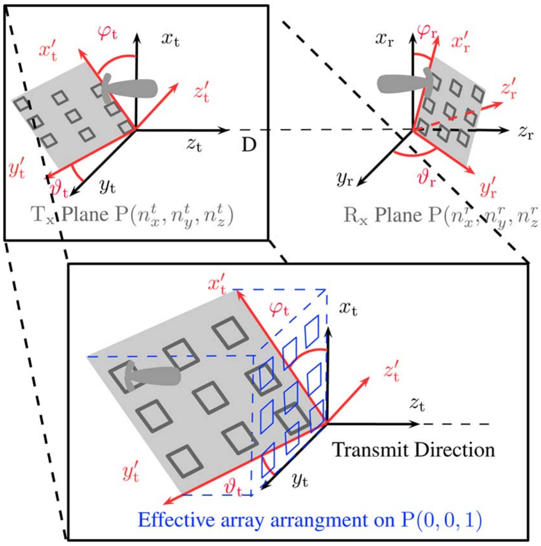

Fig. 3. Oblique UPA. Two parallel Cartesian coordinate systems are located at $\mathrm{T_x}$ and $\mathrm{R_x}$ with origins at the bottom right of the $\mathrm{T_x}$ UPA and bottom left of the $\mathrm{R_x}$ UPA. The z-axes are both in direction from $\mathrm{T_x}$ to $\mathrm{R_x}$.

of the $x'_iy'_i$-plane satisfies $\mathbf{n}_{x'_iy'_i} = \mathbf{Q}_i \cdot (0,0,1)^T = (\cos\vartheta_i \cdot \sin\varphi_i, \sin\vartheta_i, \cos\vartheta_i \cdot \cos\varphi_i)^T$. Therefore, the ϑ_i and φ_i can be obtained via

$$\vartheta_i = \arcsin n^i_{\mathrm{y}}, \quad \varphi_i = \arctan \frac{n^i_{\mathrm{x}}}{n^i_{\mathrm{z}}}. \tag{18}$$

Therefore, the plane $\mathrm{P}(n^i_{\mathrm{x}}, n^i_{\mathrm{y}}, n^i_{\mathrm{z}})$ can be formulated as

$$\cos\vartheta_i \cdot \sin\varphi_i \cdot x + \sin\vartheta_i \cdot y + \cos\vartheta_i \cdot \cos\varphi_i \cdot z = 0. \tag{19}$$

The parallel URA solutions, as indicated in Equation (2), are located on plane P(0, 0, 1) with normal vector $\mathbf{n}_{xy} = (0,0,1)^T$. The coordinates of the antenna element (m_v, m_h), $m_v \in \{0, \cdots, M_{\mathrm{V}}-1\}$, $m_h \in \{0, \cdots, M_{\mathrm{H}}-1\}$ on P(0, 0, 1) are

$$(x_i, y_i)^T_{m_v,m_h} = (d_{i,v} \cdot m_v, d_{i,h} \cdot m_h)^T. \tag{20}$$

By mapping the solution back from P(0, 0, 1) to $\mathrm{P}(n^i_{\mathrm{x}}, n^i_{\mathrm{y}}, n^i_{\mathrm{z}})$ according to Equation (19), the transceiver coordinates in Cartesian coordinate system (x_i, y_i, z_i) become

$$\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}_{m_v,m_h} = \begin{pmatrix} d_{i,v} \cdot m_v \\ d_{i,h} \cdot m_h \\ \frac{\cos\vartheta_i \cdot \sin\varphi_i \cdot m_v \cdot d_{i,v} + \sin\vartheta_i \cdot m_h \cdot d_{i,h}}{-\cos\vartheta_i \cdot \cos\varphi_i} \end{pmatrix}. \tag{21}$$

Thus, the coordinates of the antenna elements on the targeting $x'y'$-plane are

$$\begin{aligned}(x'_i, y'_i, z'_i)^T_{m_v,m_h} &= \mathbf{Q}_i^{-1} \cdot (x_i, y_i, z_i)^T_{m_v,m_h} \\ &= \begin{pmatrix} \frac{\cos\vartheta_i \cdot d_{i,v} \cdot m_v + \sin\vartheta_i \cdot \sin\varphi_i \cdot d_{i,h} \cdot m_h}{\cos\vartheta_i \cdot \cos\varphi_i} \\ d_{i,h} \cdot m_h / \cos\vartheta_i \\ 0 \end{pmatrix}. \end{aligned} \tag{22}$$

Equation (22) shows that the antenna arrangement should not always be rectangular to achieve the optimal multiplexing gain in strong LOS MIMO channel. If the antenna planes are not orthogonal to the transmission direction, the optimal antenna arrangements should be uniform parallelogram array rather than URA.

V. Conclusion

In this work, we show that the capacity of a strong point-to-point LOS MIMO channel is invariant with respect to the third dimension of the antenna structure, if the antenna size is negligible in comparison to the transmit distance. The aperture of the array in broadside relative to transmit direction determines the performance and the third dimension of the structure does not influence the antenna design. This proof brings two other interesting conclusions. Firstly, the optimized solutions of a 2D LOS MIMO array are also the best solutions for point-to-point 3D LOS MIMO systems. Secondly, the optimal antenna arrangement solutions on any curved surfaces can be easily obtained via mapping broadside solutions to the surfaces. As an example, 1D or 2D antenna arrangements that compensate the degradation from the tilt angles are discussed and represented.

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