Exploiting Road Network Properties in Efficient Shortest-Path Computation

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Abstract

The essential elements of any navigation system are a shortest-path algorithm and a map dataset. When seen in the light of the basic requirement of such a system, to provide high quality navigation solutions fast, algorithms have to be efficient and road networks have to be up-to-date. The contribution of this work is two-fold. First, the HBA* algorithm, an efficient shortest-path algorithm, is presented that mimics human driving behavior by exploiting road network hierarchies. HBA* is a fast algorithm that produces high quality routes. Second, in a thorough performance study dynamic, travel times are introduced to replace the unreliable static speed types currently used in connection with road network datasets. Dynamic travel times are derived from large quantities of historic vehicle tracking data. The integrated result, fast algorithms using accurate data, is empirically evaluated using actual road network datasets and related dynamic travel time data.

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1. INTRODUCTION

"Time is money" becomes a very concrete and meaningful statement when considering it in the context of routing and navigation applications. A quote from the field of transportation logistics suggests that saving 5% in transportation time translates to 25% added profit![1] This fact should be a more than adequate motivation for the development of methods that *efficiently* support the computation of dependable routing solutions.

The essential components of a navigation system are (i) a shortest-path algorithm and (ii) a map dataset. The delivered solution can be the shortest in terms of either distance or time. We concentrate on the latter. In order to deliver high quality routing solutions to users fast, algorithms need to be efficient and road networks up-to-date. We introduce the HBA* algorithm, a *bidirectional* version of the A* algorithm that *utilizes road network hierarchies* to achieve faster computation times. The HBA* uses *hierarchical jumping*, a technique that favors the use of the higher category roads to reduce the overall search space and to significantly improve the running time of shortest-path computation. We show that provided the road network fulfills certain properties (connectedness), the HBA* is guaranteed to find a solution. The question that needs to be answered is how good the result will be when compared to an optimal shortest-path solution.

Our expectations are that HBA* will provide superior performance in terms of computation time when compared to a classical benchmark such as the A* algorithm [12]. Our comprehensive performance study will show that while the running time can be dramatically improved, one needs to carefully tune the HBA* to achieve comparable shortest path solutions. The *performance* of the HBA* is investigated using road networks and respective travel time datasets. An important issue for routing tasks is the unreliable travel time database associated with the underlying road network. Typically, only static weights as derived from road categories and speed limits are used to calculate the fastest or shortest path for a given trip. In this work, we use *dynamic travel times* [17, 18] to assess the performance of our shortest-path algorithm (i) in relation to existing solutions (HBA* vs. A*) as well as (ii) to assess the effect of the dynamic travel times on the quality of routing solutions. Dynamic travel times are derived from vehicle tracking data also commonly referred to as floating car data (FCD) or probe vehicle data (PVD). Large collections of such data are used to derive *trends in the travel time behavior* for road networks. To compute shortest-path solutions, dynamic travel times allow us to find more accurate solutions when compared to static weights as provided by map data vendors. For this study, large amounts of historical FCD have been collected over a period of 2 years for the road networks of Athens, Greece and Vienna, Austria.

In relation to shortest-path computation, we can cite the following literature. Bidirectional heuristic search algorithms were introduced early to speed up computation in the general field of artificial intelligence [19, 22]. Many approaches have been proposed since then, among them a recent one based on the avoidance of repetitive searching usually induced by a bidirectional A* algorithm [27]. A comprehensive survey on the use of heuristics in the domain of transportation applications can be found in [11]. Introducing hierarchies in data structures as a divide and conquer strategy has been used for decades in data-intensive applications to speed up computing, especially in geospatial applications (map storage, map visualization, etc.). In the case of road datasets, it was introduced rather late (see e.g., [24, 5]). Virtual shortest-path-view was introduced in [21]. Hierarchical search has been also applied to the gaming context [1], by using hierarchical clusters of the game world and pre-computed paths between them to speed up routing. Closer related to our work, [15] uses knowledge on the road types to subdivide the road network and optimize shortest-path search. Overall, the our work is to the best of our knowledge the first to introduce a parameterizable shortest-path algorithm that combines bidirectional search with exploiting road network hierarchies and provides a comprehensive theoretical as well as empirical analysis of the algorithm and its properties.

The remainder of the paper is organized as follows. Section 2 discusses hierarchical road networks and introduces the HBA* algorithm. Section 3 describes the data scenario used in the experimentation. Specifically, it showcases dynamic travel times, an important property of the road network that directly affects shortest-path computation and the quality of its solutions. Section 4 describes our experimental evaluation using actual road networks of and related travel time datasets of Athens, Greece and Vienna, Austria. Finally, Section 5 draws our conclusions and gives directions for future work.

2. HIERARCHICAL ROUTING

In the following, we outline the basic principles behind shortest-path algorithms before discussing the properties of hierarchical road networks and how they can be exploited to improve algorithmic performance. The outcome of this discussion will be the HBA* shortest-path algorithm.

2.1 Algorithmic Solutions

A road network, made of links, is modeled as a directed graph $G = (V, E)$, whose vertices V represent intersections, and edges E represent links between intersections. Additionally, a real-valued weight function $w : E \rightarrow \mathbf{R}$ is given, mapping edges to weights. In the routing context, such weights typically correspond to speed types derived from road categories or based on average speed measurements. However, what is important is that such weights are static, i.e., once defined they are rarely changed.

Given a path $p = \langle v_0, v_1, \ldots, v_k \rangle$ in G, the weight of the path is the sum of the weights of its constituent edges $w(p) = \sum_{i=1}^{k} w(v_{i-1}, v_i)$. The weight $\delta(u, v)$ of a shortest path between vertices u and v is defined as:

$$\delta(u, v) = \left\{ \begin{array}{l} min\{w(p) : \ p \text{ is a path from } u \text{ to } v\} \\ \infty \end{array} \right\} \qquad (1)$$

A shortest path from u to v is any simple path p with $w(p) = \delta(u, v)$ [7].

Assume a directed graph with non-negative edge weights $w(u, v) \geq 0$ is given. The *single source shortest path problem* of finding a shortest path between a source vertex s and a target vertex t can always be solved by applying a uniform-cost all-pairs shortest paths algorithm (also known as greedy or Dijkstra's algorithm [10]). This incremental algorithm maintains a set S of vertices whose final shortest path weights from source vertex s have already been determined. It also maintains a priority queue of all vertices $v \in V - S$, ordered by the shortest path estimate $d(v)$. Repeatedly, the vertex $u \in V - S$ with the *minimum shortest-path estimate* is extracted from the priority queue, added to S, and all edges (u, v) are relaxed. This means that it is tested whether the shortest path estimate for v can be improved by considering a path through u. If that is the case, the shortest-path estimate $d(v)$ is updated (which also affects the ordering in the priority queue). The algorithm terminates when the set $V - S$ is empty, i.e., when all edges have been relaxed, or

[1]The 5% saving directly contributes towards increasing the profit margin.

when a designated target vertex has been extracted.

Although Dijkstra's algorithm is guaranteed to find the shortest path from s to any vertex u, it is an uninformed search algorithm that usually explores too many vertices that have no influence on shortest paths from s to t. This behavior can be improved by exploiting knowledge about the structure of shortest paths from s to t. In that context, informed search algorithms ("best-first search") have been proposed, among them pure heuristic-based algorithms and the A* algorithm [12], which combines Dijkstra's algorithm with a heuristic-based algorithm. The A* algorithm differs from Dijkstra's algorithm in that for the selection of $u \in V - S$, it uses the cost $d(u)$ of a shortest path from s to u, in combination with the *estimated cost to the goal*, $h(u, t)$. More precisely, the order of the priority queue storing $V - S$ (the "open list") is based on

$$f(u) = d(u) + h(u, t) \tag{2}$$

The value of the *heuristic function* $h(u, t)$ has to be heuristically determined. h is called *admissible* if $h(u, t) \leq h^*(u, t)$ for all $u \in V$, where $h^*(u, t)$ is the cost of a shortest path from u to t. In other words, h is admissible if it never overestimates the real cost of reaching the target. It has been shown [16, 9] that for admissible h, the A* algorithm correctly computes the shortest paths. That means, when the algorithm terminates, $d(t)$ is the correct weight of a shortest path from s to t. For shortest path problems in the Euclidean plane, the straight-line (Euclidean) distance is admissible and easy to compute, and is therefore often used as the heuristic function.

Although the A* algorithm is optimal (cf. Section 2.1) in that, if it uses an admissible heuristic function it finds the shortest path, a point of criticism is still its efficiency. We introduce in the following the hierarchical, bidirectional A* (HBA*) algorithm to efficiently compute short path solutions. The HBA* is a *bidirectional algorithm exploiting road category metadata* to effectively reduce the size of the road network during the search.

In the following, we first discuss hierarchical road networks, then introduce the actual HBA* algorithm, and finally show some of its important properties.

2.2 Hierarchical Road Networks

Roadmap data available from vendors usually provides road category information for each road segment/link. Table 1 (cf. [23]) shows a typical example of road categories, including categories such as "Freeway", "Major Road", and "Local Road of Minor Importance". In this categorization, low numbers are assigned to higher road categories, i.e., the highest road category is "0: Freeway", and the lowest "8: Other Road". Also shown are the link-based speed types (static weights), i.e., the average speed that can be achieved in a road category. This table also gives the size of the respective road networks in number of links per road category (see Figure 5 for a visualization of the network).

This road category information gives rise to interpret the network as a *hierarchical road network*: Level L_i of the road network consists of all road segments of road categories $j \leq i$, including all nodes incident to those segments. Let $G = (V, E)$ be the whole road network with vertex/node set V and road segment/link/edge set E, and let $L_i = (V_i, E_i)$. Then $V_i \subseteq V_{i+1}$ and $E_i \subseteq E_{i+1}$ for all i, and $V = \cup_i V_i$ and $E = \cup_i E_i$.

Connectivity assumption: It is commonly assumed that each level $L_i = (V_i, E_i)$ in a hierarchical road network is *strongly connected*. This means that for every $u, v \in V_i$ there exists a path in L_i from u to v as well as a path from v to u (the two paths do not have to be the same). We call a hierarchical road network that fulfills this condition *strongly connected*. We will use this assumption in Section 2.4 to prove important properties about the proposed HBA* algorithm. To check whether the road networks used in our experiments (cf. Section 4) fulfill the above assumption, we use the JGraphT library [14] implementation of an algorithm presented in [7]. The algorithm is a special application of DFS with a running time of $O(V + E)$.

Figure 1: A typical route for driving from one neighborhood to another

To best understand the significance of hierarchical road networks, consider the typical example of how roads of varying importance would typically be used in a *routing task by a human*. Figure 1 gives an example of driving from one neighborhood in Athens (Kallithea) to another (Moschato). The route length is 3.45km. The route consists of links of varying categories shown in Table 1. The route starts at level 6 (gray), and continues on levels 5 (red), 3 (blue), 4 (green), 5 (red) and arrives at level 6 (gray). Such a route represents a typical behavior of a driver searching for a route between two locations: First, she searches for a major road connecting the two areas of interest, and then she finds access roads to those major roads [2].

Table 1: Tele Atlas road categories, speed types, and size of the Athens and Vienna road network

Cat.	Description	Speed [km/h]	Athens [links]	Vienna [links]
0	Motorway, Freeway, or Other Major Road	90	1127	1374
1	Major Road Less Important than a Motorway	70	102	805
2	Other Major Road	45	4528	4535
3	Secondary Road	35	1977	4494
4	Local Connecting Road	25	13928	8045
5	Local Road of High Importance	20	7190	3262
6	Local Road	15	18042	16029
7	Local Road of Minor Importance	15	158782	35283
8	Other Road	15	1045	1495
	Total		**206721**	**75322**

The basic question that needs to be addressed is *how we can mimic such route finding behavior in shortest-path search algorithms*.

2.3 Hierarchical Networks and Shortest-Path Algorithms

Exploiting road network hierarchies in a shortest-path algorithm

requires some re-thinking of the general algorithmic strategy. The A* algorithm computes a shortest path from a source to a target by expanding nodes and recording the respective path cost. Considering the example shown in Figure 1, ideally, shortest-path search should disregard lower category links during the middle portion of the route search, since they are unlikely to contribute to a better route solution. Note that in the example of Figure 1, the *spatiotemporal category sequence* of road categories of the links along the path from the source to the target is $6-5-3-4-5-6$. The numbers in this category sequence are first monotonically decreasing, and then monotonically increasing. In other words, this sequence is *decreasing-increasing bitonic*, or short *bitonic*.

2.3.1 Hierarchical Jumping

In order to utilize the hierarchical road network, our HBA* algorithm will only consider paths that have a bitonic category sequence. The HBA* algorithm will consist of two separate A* algorithms that each compute shortest paths with a *monotone* category sequence.

Let's first assume that for every vertex $v \in V$, we would like to compute the shortest path from s to v among all paths with monotone increasing category sequence. The A* algorithm is iterative, and stores for every vertex v an implicit representation of a current shortest path from s to v with weight $d(v)$. We enforce the monotonicity constraint during the A* algorithm by storing for each vertex $v \in V$ its *current category* $cat(v)$, which is defined as the category of the edge of the current shortest path ending in v. During node expansion for a node u, only those edges (u, v) are considered for which $cat(u, v) \leq cat(u)$, where $cat(u, v)$ is the road category of this edge. The road category for v is then computed as $cat(v) \leftarrow cat(u, v)$. See Figure 4 for one iteration of this monotone A* algorithm. This simple modification of the A* algorithm ensures that only monotone sequences are considered. When $cat(u, v)$ is strictly less than $cat(u)$, this means that the routing algorithm jumped to a higher level in the hierarchical road network, and starts disregarding lower category links. Once having jumped to a higher category (i.e., a smaller number), the algorithm now stays at this level for this specific path that it explores. Note that the values of the current categories $cat(v), cat(v')$ for different vertices v, v' can be completely different, even if both vertices are incident to u via edges (u, v), (u, v'). This is because the current road category is *not* a global condition for all vertices in the open list, but a local condition for each vertex.

The advantage is that since we effectively reduce the overall size of the road network with each hierarchical jump, fewer nodes need to be subsequently explored and the performance of the algorithm in terms of memory consumption and computation speed is dramatically improved. Figure 2 illustrates the number of nodes existing for a route similar to Figure 1 when performing hierarchical jumps. During the middle portion of the search, the number of nodes that have to be evaluated is dramatically reduced. This is further evident when examining the number of nodes on a per-category basis of a road network. 70% and 50% of all nodes in the case of Athens and Vienna, respectively belong to the lowest category (local road of minor importance)! Thus, by eliminating this portion of the road network the performance of a shortest-path search will be considerably improved.

2.3.2 Hierarchical Bidirectional A* (HBA*)

We propose a new variant of a bidirectional A* algorithm to compute a shortest path from s to t that has a bitonic category sequence. This can be done in a quite straight-forward way by running two monotone A* algorithms in parallel: We simultaneously initiate a

Figure 2: Road network hierarchy sequence

monotone increasing A* algorithm from s to t (the "front search"), as well as a monotone increasing A* algorithm from t to s (the "back search"). The two solutions will meet at some high level link in the "middle" of the computed route.

The HBA* algorithm is given in pseudo-code notation in Figures 3 and 4. Figure 3 initializes the respective data structures such as the open and the closed lists, path costs, and link categories. This function initiates the two respective A* searches and checks whether any search has terminated, upon which it reconstructs the resulting shortest path. The two respective searches are coordinated in that after execution of a single node expansion it is checked whether both algorithms still operate on the same link level. A search that has reached a higher level is halted until the other search has caught up.

One iteration of the monotone A* search algorithm is detailed in Figure 4. Each execution of this function only evaluates one node from the open list. The essential task of this algorithm is to extract the node with minimal $f(x)$ from the open list and expand it, i.e., check all neighboring links and place the current node on the closed list. This algorithm terminates once it finds a node from the closed list of the other search or reaches the target node (line 4). In line 6, a specific node is expanded by retrieving all neighboring nodes that can be reached through links that are at least of the category the current node was reached by. Note that for the back search, links have to be processed in the reverse direction. To provide a concise notation, this detail has been omitted in the pseudocode description of the algorithm.

The function StepCost(x, y) returns the travel time associated with the link that connects the two nodes x and y, and $category(x, y)$ is the category of the link (x, y). The heuristic function $h(x, y)$ needs to return the estimated travel time from node x to y. In our implementation, $h(x, y)$ is defined as the Euclidean distance between x and y, divided by the average speed of the network. The right choice of an average speed is critical since a small speed would overestimate the cost to the goal and thus eliminate valuable candidate solutions. Choosing a high speed underestimates the cost and thus keeps many candidate solutions, however possibly including too many unlikely routes that may inflate the size of the open list. For our setting, our choice of an ideal speed was 110km/h, which is less than the typical speed on highways but well above the speed encountered in inner-city routes. The function Path(s, m) extracts the computed shortest path from s to m using the predecessor array P. Proper concatenation of the two extracted paths from the front and the back searches (line 17 in Figure 3) yields a path with bitonic category sequence.

```
HBAStar(s, t)
    ▷ Initialize front search and back search
 1  CL_F, CL_B ← ∅ ▷ Closed lists
 2  OL_F, OL_B ← ∅ ▷ Open lists
 3  D[s], D[t] ← 0
 4  OL_F .insert(s, D[s] + h(s, t))
 5  OL_B .insert(t, D[t] + h(t, s))
 6  P[s], P[t] ← NULL ▷ Predecessor array, implicitly storing
                      ▷ the shortest paths tree
 7  CAT[s], CAT[t] ← ∞ ▷ Current category of nodes
 8  cat_F, cat_B ← ∞ ▷ Current category of search

    ▷ Start two-sided A* search
 9  m = NULL ▷ Node at which searches meet
10  while (front and back search are active)
11     if (cat_F ≥ cat_B and front search active)
12           m = AStar(s, t, OL_F, CL_F, CL_B, cat_F)
13     if (cat_B ≥ cat_F and back search active)
14           m = AStar(t, s, OL_B, CL_B, CL_F, cat_B)
15  ▷ Searches from both sides have terminated
16  if (m ≠ NULL)
17     return res = Path(s, m) ∘ Path(m, t)
18  else return NULL ▷ no path found
```

Figure 3: HBA* - control algorithm

```
AStar(s, t, OL, CL, CL', cat)
 1  if OL ≠ ∅
 2     x ← OL .removeMin() ▷ Node with smallest f-value
 3     CL .insert(x)
 4     if x = t or x ∈ CL'
 5           return x
 6     for each y ∈ Adj[x] with category(x, y) ≤ CAT[x]
           ▷ Link is of current category or better
 7           cost ← D[x] + StepCost(x, y)
 8           if (y ∉ OL and y ∉ CL) or (cost < D[y])
               ▷ Update y
 9              D[y] ← cost
10              CAT[y] ← category(x, y)
11              P[y] = x
12              cat ← min(cat, CAT[y])
13              if y ∈ OL
14                    OL .decreaseKey(y, D[y] + h(y, t))
15              else if y ∈ CL
16                    CL .remove(y)
17                    OL .insert(y, D[y] + h(y, t))
18              else
19                    OL .insert(y, D[y] + h(y, t))
20  else return NULL
```

Figure 4: One iteration of the monotone A* - search algorithm

2.4 Algorithm Properties

In this section we show that given an admissible heuristic function and a strongly-connected hierarchical road network, the HBA* algorithm (see Figures 3 and 4) always terminates returning a bitonic path between source $s \in V$ and target $t \in V$, whose decreasing and increasing portions are each shortest.

THEOREM 1. *If the HBA* algorithm uses an admissible heuristic function h, then for any $s, t \in V$, the algorithm computes a decreasing-increasing bitonic path from s to t, whose decreasing portion is shortest and whose increasing portion is shortest as well.*

PROOF. The monotone A* algorithm (see Figure 4 for one iteration of it) by construction considers all monotone decreasing paths in G. The monotone A* algorithm basically equals the A* algorithm, with the only change being the category condition in line 6 and the addition of line 10 to update the current category of a node (see Figure 4); note that line 12 is only needed for coordinating the front search and back search. From the correctness and other related properties of the A* algorithm for admissible h [16, 9] follows that, at any time during the monotone A* algorithm with admissible h, if u is in the closed list then $d(u)$ equals the weight of a shortest monotone decreasing path from the source to $u \in V$. (If no such path exists, then the weight is considered to be ∞.)

The condition in line 4 of the monotone A* algorithm (see Figure 4) ensures that the HBA* algorithm stops with a found path iff a vertex m is found that is contained in both closed lists for the front and the back monotone A* searches. Hence, HBA* computes a path from s via m to t, such that the path from s to m is the shortest monotonically decreasing path from s to m, and the path from m to t is the shortest monotonically increasing path from m to t. □

Note that in theory it is possible to construct hierarchical road networks for which no bitonic path exists between two given vertices s and t. However, in Theorem 2 below we show that for hierarchical road networks that fulfill a reasonable connectivity assumption, which we introduced in Section 2.2, a bitonic path always exists for any choice of s and t, and hence HBA* always terminates finding a path.

THEOREM 2. *If each level in a given hierarchical road network $G = (V, E)$ is strongly connected, then for any choice of $s, t \in V$ there exists a decreasing-increasing bitonic path from s to t in G.*

PROOF. Let $L_0, \ldots, L_k$ be the levels of the hierarchical road network, with $k \geq 0$, and $L_i = (V_i, E_i)$. We show the claim by induction on k. Since L_0 is strongly connected, the category sequence of any path in L_0 between any two vertices $s, t \in V_0$ is constant $0, 0, \ldots, 0$, and hence trivially bitonic. Now assume the claim is true for k (inductive hypothesis), then it remains to show that the claim is true for $k + 1$. Let $s, t \in V_{k+1}$ be any two vertices. Remember that by definition, $V_k \subseteq V_{k+1}$. If both $s, t \in V_k$, then the claim follows by the inductive hypothesis. Now assume $s \notin V_k$ or $t \notin V_k$ (or both), and let $\pi : s = v_0, v_1, \ldots, v_{l-1}, v_l = t$ be a path in V_{k+1} from s to t. If the category sequence for π is constant with category $k + 1$, then it is also bitonic. Now assume the category sequence is not constant. Let (v_i, v_{i+1}) be the first edge in π with category less than $k + 1$, and let (v_j, v_{j+1}) be the last edge in π with category less than $k + 1$. This implies that $i <= j$, and $v_0, \ldots, v_i$ and $v_j, \ldots, v_l$, if non-empty, have constant category sequences with category $k + 1$. (Note that one of these sequences may be empty, but not both.) By construction, $v_i, v_j \in V_k$, and by the inductive hypothesis there exists a path π' in L_k from v_i to v_j with bitonic category sequence, starting and ending with at most k. Therefore the concatenation $s = v_0, \ldots, v_i \circ \pi' \circ v_j, \ldots, v_l = t$ has a bitonic category sequence as well. □

Although it is not clear at first whether the front and the back search in HBA* algorithm have to meet, Theorem 2 shows that provided the hierarchical road network is strongly connected, HBA* is guaranteed to terminate with a found path. This *connectivity assumption* holds typically for any commercially available map dataset, i.e., connectivity at various levels of the road network hierarchy is a property guaranteed by the map data vendor. However, for all road network datasets used in our experiments it is certain that they fulfill this property (cf. Section 2.2).

Theorem 1 shows that if an admissible heuristic function is used, the algorithm is guaranteed to find a bitonic path that is optimal in the sense that the decreasing portion and the increasing portion are each optimal. Note that for road networks with travel time edge weights it is not obvious how to define a good heuristic function that is provably admissible (except for the trivial $h(u,t) = 0$). The conventionally used Euclidean distance does not yield a meaningful heuristic function in this case, since it does not encode time information. In our implementation we do however integrate the Euclidean distance, and compute $h(u,t)$ as the Euclidean distance divided by speed, see Section 2.3.2. There is however no provably correct value for the speed, as it could theoretically be arbitrarily large, and hence distance/speed arbitrarily small. For our applied setting, our choice of an ideal speed was 110km/h, which is less than the typical speed on highways but well above the speed encountered in inner-city routes.

Our HBA* algorithm, given an admissible heuristic function, computes a bitonic path with shortest decreasing and increasing portion but it does not necessarily compute the overall shortest bitonic path as it does not optimize over all possible split vertices. By running both the front search and the back search until all vertices have been discovered, an optimal split vertex and hence a shortest bitonic path can be computed. This, however, would not yield the desired speedup in runtime. Hence, we decided to opt for finding one bitonic path with shortest decreasing and increasing portion. The condition in lines 11 and 13 of the HBA* (see Figure 3), attempt to keep a balance between the decreasing and increasing portion of the computed path.

3. DATA

Road networks and related travel time data are an essential aspect for any meaningful route computation. The following sections discuss the specific datasets used to assess the performance of the HBA* algorithm.

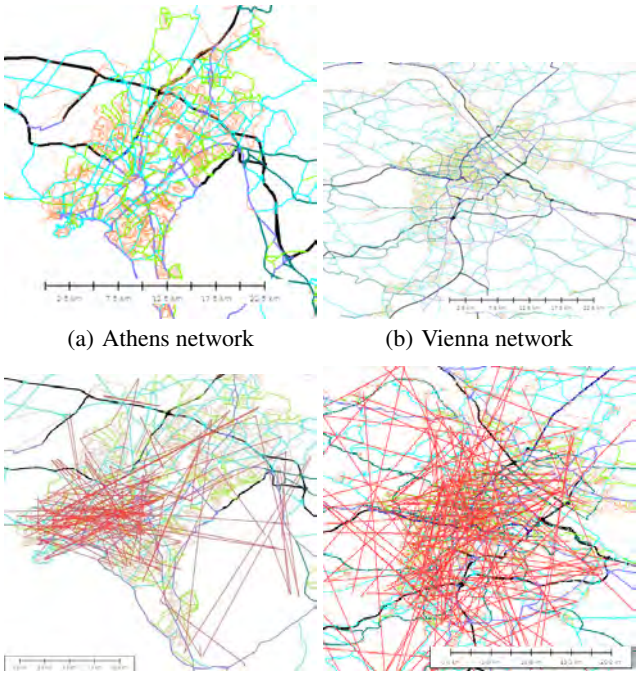
3.1 Road Networks

In terms of road network, our experiments focus on the greater metropolitan areas of Athens, Greece and Vienna, Austria. The portion of the road network that was used has an extent of roughly 25×25km in each case. The road networks are visualized in Figures 5(a) and 5(b). Major roads are indicated by darker colors.

Table 1 lists the road categories and respective size in terms of number of edges (links) for each road network. Table 3 presents an aggregate view showing total size vs. number of links belonging to major road categories. This distinction is important as we will see in the following that dynamic travel times will only be available for major road categories. An interesting observation is that although the Athens road network is three times the size of Vienna, the number of roads belonging to major road categories is almost the same!

3.2 Travel Times

To improve the quality of shortest-path solutions, one needs to carefully select the underlying weight database of the road network



(a) Athens network (b) Vienna network

(c) Athens queries (d) Vienna queries

Figure 5: Vienna and Athens road networks

graph $DB(w(u,v))$ (cf. Section 2.1). Typically, weights in map data are static and correspond to link-based speed types, which are derived from the respective road category and its associated speed limit, or a speed type determined by costly road-side surveys.

This section introduces a dynamic weight database in which the travel time associated with a link changes based on a temporal argument (speed profile). The idea is to derive dynamic weights from historical traffic assessment based on sensor measurements in the form of FCD. Using the causality between historical and current traffic conditions, weights in the form of *dynamic travel times* will replace static weights.

With the availability of cheap positioning technology and the penetration of asset tracking applications such as fleet management applications, vehicle tracking data, as a component of *Floating Car Data* (FCD), becomes an important tool for traffic assessment and prediction. FCD refers to using data generated by one vehicle as a sample to assess to overall traffic conditions ("cork swimming in the river"). Having large amounts of vehicles collecting such data for a given spatial area such as a city (e.g., taxis, public transport, utility vehicles, private vehicles) will create an accurate picture of the traffic condition in time and space [20, 17].

To derive dynamic travel time datasets from collected FCD, map-matching algorithms are needed, as they relate GPS tracking data to the road network [3, 26]. The resulting travel times then need to be aggregated using a data warehouse architecture such as described in [17, 18]. Existing commercial solutions employ similar approaches. For example, Dash [8] markets off-board navigation devices that provide online traffic information based on the submitted FCD from the Dash device network. Recently, similar to Dash, TomTom introduced IQRoutes [25], a routing engine that relies on offline speed profile information derived from collected tracking data.

To illustrate the power of dynamic travel times, consider the ex-

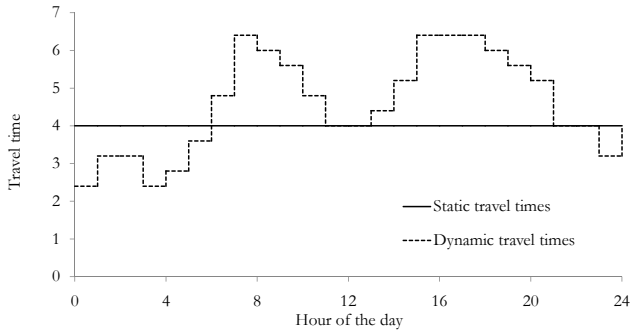


Figure 6: Speed profile

Table 2: Amounts of collected FCD for Athens

		FCD	Travel Times
Athens	Daily	46365	73523
	2 years	20M	28M
Vienna	Daily	481479	718408
	2 years	232M	320M

ample of Figure 6 showing varying travel times (in minutes) for a road segment. The figure shows a step function giving different travel times for each hour of the day and a static travel time remaining constant throughout the day. This shows the potential of dynamic travel times to improve the weight database and in turn increase the quality of routing solutions. The collection of such dynamic travel times is also referred to as *speed profile* of a road network.

Speed profiles are understood by observing travel times over a longer time period, i.e., for one year, and they are derived by aggregated individual travel times measures using meaningful temporal granularity. As travel times are recorded with respect to a *specific edge* in the road network and a specific time, travel time aggregation is achieved by computing the average of a set of travel times having similar timestamps. An example here would be to average all travel times recorded on Mondays from 9:00 - 9:15. Speed profiles represent travel time trends in the road network and can thus be used as a simple prediction mechanism. The assumption is that future travel times will be similar to travel times observed in the past.

In our case speed profiles have been compiled using FCD from a commercial vehicle fleet (Athens) and from a taxi fleet (Vienna). To illustrate the amount of data that is generated, Table 2 gives the FCD that was collected on a typical day (Feb. 4, 2008) and overall during a 2 year period. The amount of data collected for Vienna is roughly ten times the amount of the Athens data. In the former case, a taxi fleet of around 1000 vehicles is collecting the data, while in Athens only 120 delivery trucks are being tracked. Feeding this data into the map-matching algorithm produces roughly 50% more travel time data, since two consecutive GPS position samples 30s apart are usually mapped to a path in the road network that consists of more than one edge. For our experiments, *28M and 320M travel times were generated* for the respective cities.

The temporal granularity of the speed profile was 15min intervals for each weekday (cf. [18]). I.e., for each quarter of an hour during a week, the "typical" travel time for one of the 22k and 19k links belonging to major roads of the respective cities is computed.

Note that only dynamic travel times for such higher category roads (cf. Table 3) were computed due to a lack of data and random

Table 3: Dynamic Travel Time dataset for Athens

	Size road network [links]	Higher cat. roads [links]	Size TT dataset [tt entries]
Athens	207,000	22,000	7M
Vienna	75,000	19,000	6M

behavior. Typically, the FCD coverage of lower category roads is insufficient and produces inconclusive data. Our data collection as well as previous studies [4] show that it is not possible to use historical travel times to establish speed profiles for lower category roads, as the travel times fluctuate considerably due to non-traffic related circumstances such as parking vehicles, local maneuvering, pedestrians, etc. Thus, the dynamic weight dataset in the following experimentation is a hybrid one. For higher category roads dynamic travel times are used, while for lower category roads we use static travel times as indicated in Table 1.

The second weight dataset used in the experiments comprises only *static travel times*. They are derived from speed information that is part of the road network dataset. In our case, speeds range from 90km/h for inner city highways to 15km/h for local roads (cf. Table 1).

4. EXPERIMENTAL EVALUATION

The objective of our experiments is (i) to compare the performance of the HBA* algorithm in terms of quality of result and cost as well as (ii) to assess the respective impact of using dynamic travel times. To evaluate the HBA* algorithm we use a standard A* algorithm to benchmark its performance.

4.1 Tuning HBA*

The HBA* algorithm exploits an important aspect of the road network, its hierarchical structure, to improve its running time. In utilizing hierarchical jumping, the algorithm mimics human driving behavior, i.e., when given the choice, it selects higher category roads to reach a destination. On the other hand, hierarchical jumping, especially when used early on in shortest-path search may eliminate candidate solutions and provide suboptimal results. To address this potential issue, we introduce an *initialization buffer*. I.e., assume that we want to compute a shortest path from s to v, then the initialization buffer $I(\epsilon)$ around both s and v prevents the use of hierarchical jumping for all vertices
$u \in I(\epsilon) : \{dist(u, v) < \epsilon \vee dist(u, s) < \epsilon\}$.

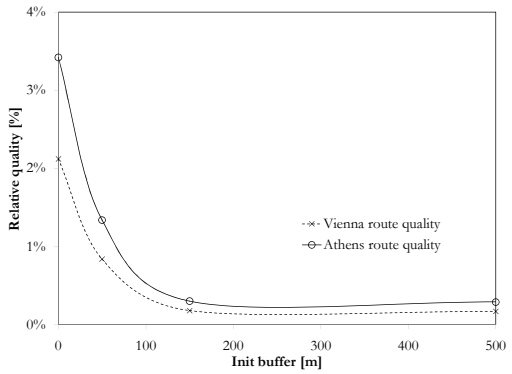
Using this approach, the HBA* essentially postpones the choice of a higher category road in search for a potentially better alternative. As we shall see in the following experimentation, this modification improves the quality of the solutions by only minimally impacting the algorithmic cost.

In our experimentation we utilized $\epsilon = [0, 50, 150, 500]m$. Figures 7(a) - 7(d) show the *relative results* when comparing the performance of the respective four instances of the HBA* using various initialization buffers to the A* algorithm. The following sections discuss the results in detail.

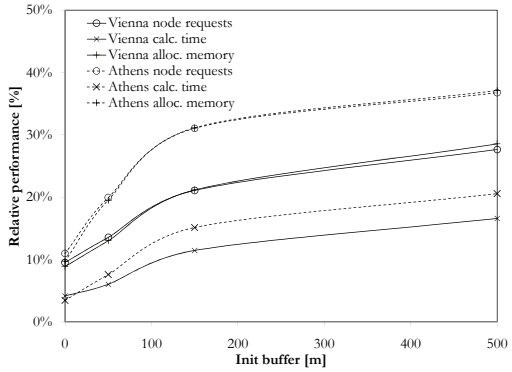
4.2 Shortest Path Quality

To assess performance, the two algorithms are compared using a set of 150 shortest path problems, each uniformly distributed over the city area. Figures 5(c) and 5(d) visualize the queries as sets of line segments connecting source and target nodes. The Euclidean distance between the nodes ranges from a couple of 100m to 20km.

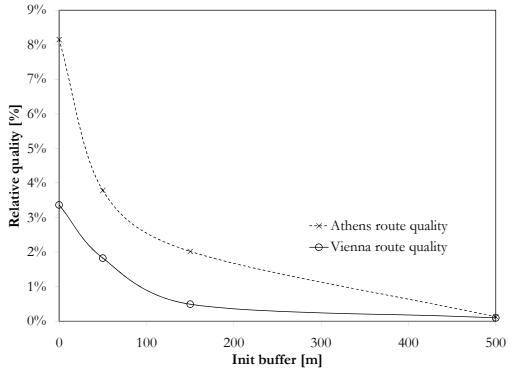
*4.2.1 Plain HBA**



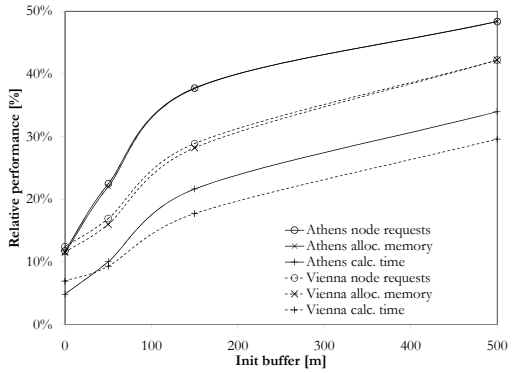
(a) Quality and static weights



(b) Cost and static weights



(c) Quality and dyn. weights



(d) Cost and dyn. weights

Figure 7: HBA* initialization buffer: shortest-path quality and cost

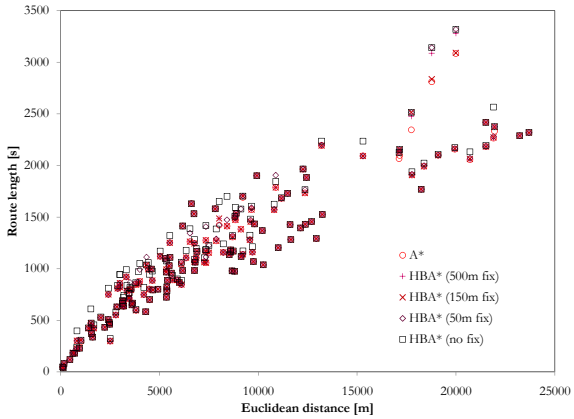


Figure 8: Shortest path results, static weights, Athens

Assuming the heuristic function $h(u,t)$ to be admissible, the A* is an optimal algorithm that is guaranteed to find the shortest path. Using the same heuristic function as defined in Section 2.3.2, Figure 7(a) shows that a plain HBA* ($\epsilon = 0$) in the case of *static weights* produces solutions that are on average 2% and 3.5% longer than those of the A* for Vienna and Athens, respectively.

Figure 8 gives absolute shortest-path results and shows for Athens (Vienna results are similar) the lengths in terms of travel time for the set of routes computed by the A* and HBA* algorithms. The travel times are plotted with respect to the Euclidean distance between the source and the target of a route. This detailed view shows that for 45% and 50% of the routes (Athens and Vienna), the two algorithms find the same solution.

It is interesting that the plain HBA* algorithm performs this well, even though it considers only a subset of possible paths (namely, bitonic paths) and it heuristically stops immediately when the front and back searches meet. This shows that coordinating the front and back searches by roughly staying on the same level of the road network (lines 11-14 in Figure 3) yields surprisingly good results.

In the case of *dynamic weights* the quality disadvantage of the HBA* increases to 3.5% and 8% respectively. The main reason for this are the lower actual travel times measured for higher category roads. While the A* is not bound to road categories, so does the HBA* get "trapped" due to hierarchical jumping on top level roads that actually exhibit low speeds (cf. Section 5 on addressing this problem). Due to a in relation to the entire network smaller number of higher category roads, this problem is more pronounced in the case of Athens.

4.2.2 *Using Initialization Buffer*

The quality of the HBA* can be improved dramatically when using the initialization buffer. Figure 7(a) shows that for static weights and, e.g., $\epsilon = 150m$ the quality of the shortest-path solution is within 0.2% of that of the A* algorithm. Examining the specific results for Athens in detail (Figure 8), shows that on an individual route basis, the gap between A* and HBA* closes with an increasing initialization buffer.

For the case of dynamic weights and for $\epsilon = 150m$, the quality improves to 0.5% and 2% for Vienna and Athens, respectively. While, increasing the initialization buffer to $500m$ only marginally improves the quality for Vienna, it however does improve the quality for Athens to within 0.2%. Judging by the quality of the shortest-path results, the initialization buffer proves to be a useful means for tuning the HBA* algorithm. It will be interesting to see how it af-

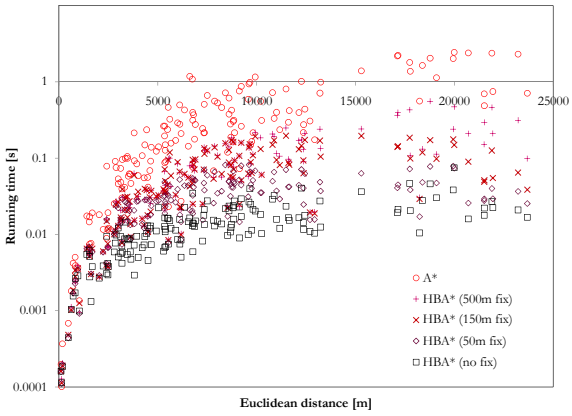


Figure 9: Running time, static weights, Athens

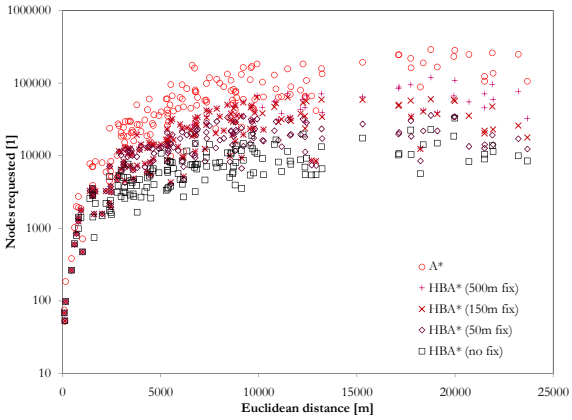


Figure 10: Requested nodes, static weights, Athens

fects the computation cost.

4.3 Computation Cost

A major advantage of the HBA* algorithm is its improved running time due to the fact that it evaluates much fewer nodes of the road network graph when computing a shortest path.

It is evident from Figure 7(b) that the plain HBA* algorithm computes a shortest path 20 times faster than the A* algorithm (less than 5% of its running time). Figure 9 provides more detail for the Athens dataset. A typical run of the HBA* algorithm takes in the range of 0.0001s (short routes) to 0.01s (long routes), while the running time of the A* algorithm may be up to 3s.

The quintessential reason for this running time difference of the algorithms can be found in the number of nodes processed in each case. Figure 10 shows that the A* algorithm processes up to ten times more nodes than the HBA*. Consider here the example of the 15km Euclidean distance route in the middle of the chart. The A* processes 120,000 nodes, whereas the HBA* only evaluates 12,000 nodes. The same fact is evident from Figure 7(b) showing relative cost figures. The plain HBA* ($\epsilon = 0m$) accesses only 10% of the nodes of the A* algorithm.

The quality of the shortest-path solution can be improved by increasing initialization buffer. Figure 7(b) shows that the running time of the algorithm increases as well. For the case of $\epsilon = 150m$, which yields results comparable to the optimal A* solutions, the running time more than doubles. However, what is important is that the running time of the HBA* is still only 10% that of the A*, thus being *faster by an order of magnitude*.

When developing algorithms, an important characteristic is the *size of the allocated data structures* during the execution of the algorithm. The A* algorithm maintains a closed and an open list, which are both implemented as heaps. The HBA* maintains two sets of these data structures. Figure 7(b) shows that the relative allocated memory is 10% that of the A* for the case of the plain HBA* and for $\epsilon = 150m$ increases up to 20% and 30% for Vienna and Athens, respectively. In absolute numbers, the open list of the HBA* is for all searches around 50 nodes. This small size is due to the hierarchical jumping and its effect on node expansion, i.e., highways do not have intersections, thus only one node is typically expanded and the open list essentially does not grow. Here, in contrast the A* open list reaches a size of up to 1000 nodes.

While, the discussion of cost figures so far only concerned static weights, the respective trends are quite similar for the case of dynamic weights (cf. Figure 7(d)).

4.4 Static vs. Dynamic Travel Times

Having two alternative shortest-path algorithms, each with its respective strength, the following section assesses qualitative improvements to algorithmic solutions *utilizing dynamic travel times*. As such, this section goes beyond evaluating the respective algorithms but assesses the potential of dynamic weights for improving routing solutions. In the specific experiments, in addition to static weights, two dynamic travel time datasets for 8h and 13h on Mondays were used. This selection was based on available travel time data (collected FCD) and expected impact on the routing solutions, i.e., common sense expectations could be that 8h constitutes rush hour, whereas 13h means smoother traffic.

For the quintessential experiment with respect to dynamic travel times, i.e., *to assess the improvement of the route quality*, we compared dynamic weights to static weights (cf. Figure 8), by recomputing for the latter the time it would take to traverse those routes using the more accurate dynamic travel times! Table 4 gives the percentage of the shortest-path solutions that were improved and the average improvement in each case. For example, for Athens, using HBA* and the dynamic travel time dataset for 8h, 55% of the 150 shortest-path solutions were improved by an average of 11.5%. Examining the improvements in detail shows that the improvements may range from 27% to somewhat less than 1%.

The following observations can be made from these experiments:

- the shortest-path solutions for Athens benefit more from the dynamic travel time data - this can be attributed to greater fluctuations in traffic conditions for Athens, i.e., Athens exhibits greater fluctuations in its speed profiles than Vienna (cf. Figure 6).

- a "rush-hour" effect can be observed when comparing 8h to 13h shortest path improvements. This effect is more evident in the Vienna data, e.g., in case of the A* algorithm yielding 9.1% vs. 6.1% improvement for 8h and 13h, respectively.

4.5 Summary

The HBA* algorithm computes shortest paths much more efficiently than the A* algorithm does. Utilizing an initialization buffer, (i) the quality of solutions is virtually identical to that of the A* algorithm and (ii) its running time is 10 times faster. Due to hierarchical jumping, it examines 10 times fewer nodes and also allocates smaller data structures.

While dynamic weights slightly affect the performance of the algorithms, they improve the quality of individual shortest-path so-

Table 4: Qualitative improvements using dynamic travel times

Athens

	HBA*		A*	
	affected	imp.	affected	imp.
8h	55%	11.5%	65%	11.9%
13h	54%	11.1%	60%	10%

Vienna

	HBA*		A*	
	affected	imp.	affected	imp.
8h	53%	9.1%	59%	9.8%
13h	41%	6.1%	40%	7.1%

lutions on average by 10%. Thus, when used, dynamic travel times represent a significant qualitative step for shortest-path solutions.

Overall, combining dynamic travel times with the HBA* algorithm generates a *fast and accurate solution base* for routing and navigation applications.

5. CONCLUSIONS AND FUTURE WORK

The objective of this work was to develop an efficient algorithm to compute shortest-path solutions fast and at the same time not to compromise the result quality when compared to optimal solutions. Specifically, an efficient shortest-path algorithm termed HBA* was presented. The algorithm is bidirectional and exploits road network hierarchies by using hierarchical jumping to improve on computation speed. Using an initialization buffer, a technique that postpones hierarchical jumping for early stages of the shortest-path search, the experiments showed that the HBA* computes solutions 10 times faster in comparison to the A* algorithm, while essentially producing identical results.

Another important aspect was the use of dynamic travel times in the performance evaluation. While typically static weights that are derived from road category information and edge length are used in routing tasks, we propose the use of *dynamic travel times* that are derived from historical floating car data. Dynamic travel times represent *speed profiles* that provide travel time trends based on the time of the day for a road network. In terms of routing solution, this data leads to a significant qualitative improvement, which is shown to be 10% on average, but can be up to 30% on an individual basis.

Our ongoing and future work is as follows. While this work only presents experimental results for Athens and Vienna, we are currently in the process of testing this technology with existing fleet management solutions and their respective routing engines. The objective is to develop a plug-in technology that can be used in as many dynamic travel-time contexts as possible. An aspect for improving the performance of the HBA* for dynamic travel times are the road network categories as provided by the map manufacturer (cf. Section 4.2). Dynamic travel times show that given the right conditions, high category roads have low travel times and should be avoided. While the A* does not rely on category information, the HBA* is restricted by it due to hierarchical jumping. Modifying category information based on dynamic weights could in this case improve the algorithmic performance.

Acknowledgment

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