Temporal Reasoning with Intervals in Branching Time

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Abstract

Allen *[ALLE83]* adapted path-consistency techniques *[MACK77]* to heuristic reasoning concerning intervals over linear time, by calculating the composition table of binary relations on intervals, and using it in the path-consistency algorithm. We consider here a model of branching time which is dense, unbounded, future branching, without rejoining branches. The algorithm in *[ALLE83]* works directly with branching-time intervals, provided only that the composition table of the binary branching-time interval relations is used instead of Allen's table *[LADK88]*. Here we calculate the composition table which has to be used, which is considerably more complex than the table for linear-time intervals. This provides a heuristic, cubic-time algorithm for reasoning with branch-time intervals.

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1 Introduction

We focus in this paper on the structure of a particular branching model of time, which is suitable for path-consistency computations on time intervals [MACK77, MACK87, ALLE83]. Path-consistency was adapted from the constraint satisfaction domain by James Allen for temporal reasoning with intervals over a dense, unbounded linear time model [ALLE83]. The algebraic nature of this heuristic reasoning technique has been investigated in [LADK88]. We shall assume some knowledge of both path-consistency computations, and Allen's adaptation of them to linear-time interval reasoning.

It was shown in [LADK88] that, given a class R of binary relations which is closed under the operations of union, intersection, complement, converse and composition, (an *algebra of relations* in the sense of Tarski), one may use path-consistency algorithms, which use just the intersection and composition tables for the class of relations, for heuristic reasoning using networks of constraints. It was also shown that one may start from a class A of binary relations on intervals that are pairwise disjoint, and such that any pair of intervals is in exactly one of these relations (a class of 'atomic' relations). If the class R is finite, then one may construct all relations in R by taking all possible unions of atomic relations (i.e. all unions of relations in A). Given the table of compositions of atomic relations, one may calculate compositions of arbitrary relations in R, and therefore apply path-consistency algorithms to networks over R. This, in essence, is what was done by Allen [ALLE83] for the case of R = linear-time interval relations over a dense, unbounded linear order of points.

We first define a model of branching time suitable for our purposes. We then define a class of atomic binary relations on intervals in this model, and calculate the composition table. This information is sufficient to allow the direct use of path-consistency algorithms on networks of these relations, in the manner of Allen. In particular, if Allen's routine *Constraints* is modified to use the composition table for branching time interval relations which we calculate, instead of his table, then his algorithm *Add R(i,j)* may be used unchanged for constraint satisfaction in the branching-time interval domain. (This follows directly from considerations in [LADK88]).

The need for branching time models in artificial intelligence and other areas of computer science has been argued by others, as we note below. We do not have the space to iterate these arguments here. In the next section, we

briefly introduce constraint problems and path-consistency. Section 3 gives the underlying point model of branching time, section 4 the atomic relations, and section 5 the composition table for the branching-time interval relations.

Other Work. Several different branching temporal models have been considered for use in various different areas of computer science, in particular artificial intelligence, and concurrency theory. In a branching time structure, each point is regarded as having many different possible futures, although only one past [PNUE77, McDER82, VanB88, EMER89]. Other approaches include parallel linear "worlds" [KAHN77, SHOH88] and models based on relativistic space-time [GOLD80, RODR90a, ANGE90b]. More general models based on a partial order of events are considered in [LAMP86], [ANGE89a]. Non-linear models attempt to represent the possibility of concurrent events or the idea that there are many different ways that events could develop depending on potentially unknown circumstances.

2 Path-Consistency Computations

We give here a brief introduction to path-consistency computations as used in constraint networks. A binary CSP (constraint satisfaction problem), or BCSP, is given by a formula $(P_1(x_1) \wedge ...P_n(x_n) \wedge P_{12}(x_1, x_2) \wedge P_{13}(x_1, x_3) \wedge ..... \wedge P_{n-1,n}(x_{n-1}, x_n))$, where the P_i and P_{ij} are predicate symbols representing constraints on the variables x_i. The constraints are binary relations, i.e. sets of pairs of objects from some domain. In our case the domain is intervals over branching time. Thus we may freely use set-theoretic operations such as union and intersection on these constraints, along with other operations specific to binary relations. A BCSP is *satisfied* by finding values for the variables which satisfy the formula. The unary constraints represent domain constraints on values for each of the variables, and the binary constraints are usually of the form $P_{ij}(x_i, x_j) \equiv ((x_i R_1 x_j) \vee .....(x_i R_q x_j))$ where the R_p are a fixed collection of disjoint 'atomic' relations over the domain of interest. For examples, and bibliography, see the survey *[Mac87].*

Given binary relations $R(x, y)$, $S(x, y)$, the composition $R \circ S$ of the relations is defined by

$$(R \circ S)(x, y) \Leftrightarrow (\exists z)(R(x, z) \,\&\, S(z, y))$$

So $(a, b) \in (R \circ S)$ if and only if there is a value c such that $(a, c) \in R$ and $(c, b) \in S$. Now, suppose we have a CSP in the variables $x_1, .., x_n$, and let P_{ij} be the relational constraint between x_i and x_j. Then for any satisfying values $a_1, ..., a_n$ for the variables $x_1, .., x_n$ we must have, by the definition of composition, $\langle a_i, a_k \rangle \in P_{ik} \Rightarrow \langle a_i, a_k \rangle \in P_{ij} \circ P_{jk}$, for any $i, j, k \leq n$. This necessary condition may be used as a pruning technique to narrow down the potential choices of a_i and a_k. This observation is the motivation for path-consistency computations. A path-consistency computation consists in repeating the computation

For every triangle (i, k, j) in A: do $P_{ij} \leftarrow P_{ij} \cap (P_{ik} \circ P_{kj})$.

until for any $i, j, k \leq n$ we have $P_{ik} \subseteq P_{ij} \circ P_{jk}$. See [LADK88] for more details.

3 The Model

Among branching-time models, there are many different alternatives, corresponding to a number of fundamental choices: discrete, dense, or continuous time; finite or infinite extent; branching into the past and future or just branching into the future; branches can rejoin or remain forever separate. We chose a model which is **dense, of infinite extent, future branching, without rejoining branches**.

The branching-time model consists of a (strict) partially ordered set (T, $<$) of time points satisfying the following additional conditions. We state the conditions informally, since the formalisations are well-known.

(B1) (**Unbounded**) For all t in T there exist t_1 and t_2 in T with $t_1 < t < t_2$.

(B2) (**Density**) For all t_1, t_2 in T there exists a t in T with $t_1 < t < t_2$.

(B3) (**Future Branching**) For all t_1 and t_2 which are incomparable (t_1 is neither $<$, $>$, nor $=$ to t_2: written $t_1||t_2$) there exists a t with $t < t_1$ and $t < t_2$, but there exists no t' with $t_1 < t'$ and $t_2 < t'$.

(B4) (**Greatest Lower Bounds**) For all t_1, t_2 in T there exists a greatest lower bound $t = glb(t_1, t_2)$ in T. If $t_1||t_2$, the point t is called a **branch point**.

(**B5**) (**Dense Branching**) For any pair of comparable points $t_1 < t_2$ in T there exists a t in T such that $t_1 < t$ and $t_2||t$. (In other words, between any two points in T there exists a branch point.)

Generally, a (temporal) **interval** over T thought of as a set of points

$$[t_1,t_2] = \{\ t : t_1 \leq t \leq t_2\ \}.$$

Notice that an interval defined in this manner is a linear (totally ordered) structure.

We do not need to address questions of a topological or set-theoretic nature, however, and the algebraic structure of interval relations suitable for constraint satisfaction techniques is unchanged if we regard an interval instead as simply an ordered pair of points $\langle a, b\rangle$ with $a < b$. This abstraction allows the formulation of interval reasoning as a problem in relation algebra, following [LADK88]. We shall use the notation $[a, b]$ for the interval represented by the pair of points $\langle a, b\rangle$. Thus issues such as whether the endpoints are included, or not, have no interpretation in our formulation. However, we do consider explicitly the branch points in our enumeration of atomic interval relations. Relations in which one of the intervals starts at the branch point are distinguished from relations in which neither do. This increases the total number of atomic relations we have to consider.

4 Atomic Branching-Time Relations

Figure 1 shows the thirteen atomic binary relations between two intervals in linear time. The figure indicates that the relations can be typified by the relations ($<$, $>$, or $=$) that exist between the endpoints of the two intervals. In branching time, all these relations still pertain, but there is the further possibility that the endpoints be *unrelated*. If the two intervals are $I = [I_s,I_f]$ and $J = [J_s,J_f]$, then $I_s||J_s$ implies that the intervals are completely unrelated (a new relation, named **U**), while I_f related to J_f by $<$, $>$, or $=$ implies that one of the thirteen linear relations (defined by Allen) holds. If, however, I_s is related to J_s but $I_f||J_f$, there are ten other new relations resulting from the three choices of I_s $<$, $>$, or $=$ J_s and from the relative location of the branch point, $p = glb(I_f, J_f)$, with respect to I_s and J_s. Figure 2 presents

the eleven new atomic branching-time relations, completing the twenty-four that exist. Figure 3 illustrates these same relations graphically.

It should be remarked in passing that without the axiom (**B4**) assuring the existence of greatest lower bounds, other relations could be classified in which there is no branch point, p. If the position of the branch point is ignored and only the relationship of the endpoints considered, then only a total of seventeen (rather than 24) relations emerges.

5 Compositions of Branching-Time Relations

As we have noted, the path-consistency technique adapts directly to the branching-time relations, using just the branching-time composition table instead of Allen's table in his routine *Constraints* [ALLE83]. (This follows from considerations in [LADK88.2]). Hence one needs to have available the compositions of relations between intervals. One may do this by calculating the table of compositions of the atomic relations, and the compositions of more general relations (unions of the atomic relations) will follow from the distributive laws [LADK88.2]. In this section, we calculate the composition table, which is complex for the case of branching time, in contrast with the relatively simple linear-time case of Allen. This calculation is the main content of this paper.

The composition table is of size 24 by 24, for the twenty-four atomic binary temporal relations between two intervals over our branching-time model. Each composition is expressible as a disjunction of the atomic relations, a greater number of terms indicating greater generality. The most general relation possible is the *universal relation*, **1**, indicating that the relation is the disjunction of all atomic relations. Due to the large number of disjuncts appearing in some compositions, a number of abbreviations besides **1** are introduced in Tables 2-4. The tables become more compact and the patterns which arise are easier to discern.

The table of compositions itself is presented piecemeal. Table 1 presents the compositions of the eleven new branching-time relations with one another, while Table 2 gives their compositions with the twelve linear-time relations. Although one might imagine that the compositions of the twelve linear-time relations with themselves would remain the same as in the linear-time model, this is not the case. For instance, if i O$^\smile$ j and j O k, in linear

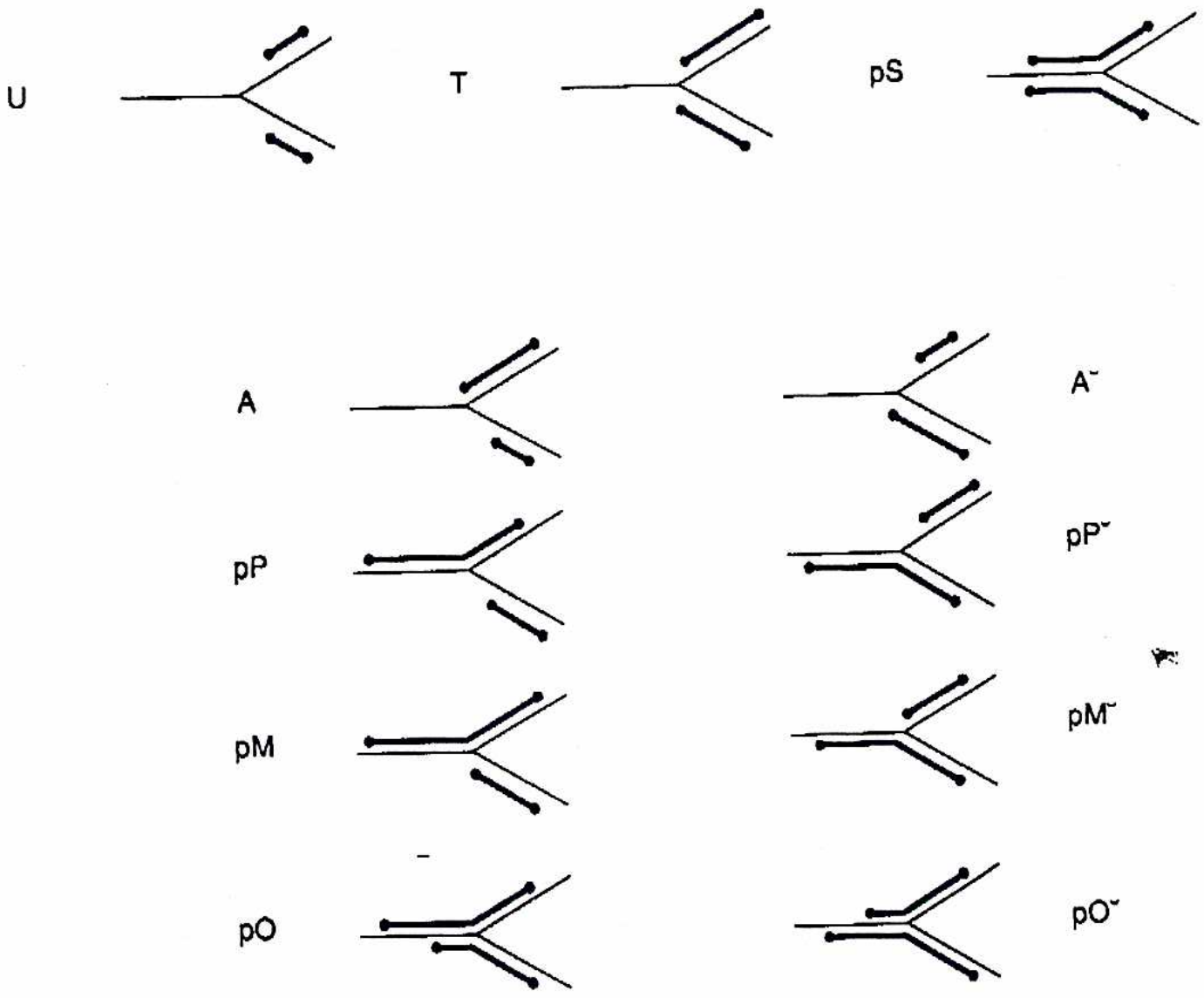

FIGURE 1.

Atomic Branching Time Relations

time it can be concluded that

$$i(< +M + O + F^{\smile} + D^{\smile} + S+ = +S^{\smile} + D + F + O^{\smile})k$$

while in branching time the same composition allows, for example, i pO k. Only the compositions of the linear-time relations which change in branching time are presented in Table 4. The rest of the table of compositions of linear-time relations may be read directly from [ALLE83] and we do not repeat it here.

To understand how the tables of compositions are obtained, it should first be observed that for any relations r and s,

$$(r\,;s)^{\smile} = (s^{\smile}\,;r^{\smile})$$

This allows almost half of the complete table to be deduced mechanically from the other half; in particular, the 12 by 11 table of compositions of linear-time relations with the branching- time relations is completely derivable from Table 2 using this identity and hence is not shown.

For an example of the calculations, in order to find A;A , suppose that $[I_s,I_f]$ A $[J_s,J_f]$ A$^{\smile}$ $[K_s,K_f]$. Then $I_s < J_s =$ p1 and $K_s < J_s =$ p2, where p1 = glb(I_f,J_f) and p2 = glb(J_f,K_f).

It follows that p1 = p2, so that glb(I_f,K_f) $\geq$ p1 and both I_s and K_s are $<$ p1. This allows all linear-time relations, and all relations which have the branch point following the start of both intervals: pO and pO$^{\smile}$.

Since it is common for a composition to include all atomic relations that satisfy some condition such as $I_s < J_s$, abbreviations are introduced for these relations. In Tables 2-4,

$[<_s\ ]$: disjunction of all relations between I and J with $I_s < J_s$ and

$\{<_s\}$: disjunction of all linear-time relations between I and J with $I_s < J_s$.

The corresponding meanings are assigned to $\{=_s\}$, $[=_s]$, $\{<_f\}$, etc. In a similar spirit, {1} is used to represent the universal relation in linear time and hence the disjunction of the thirteen linear-time relations. Finally, since relations of the form O+pO and M+pM arise frequently, the shorthand

Rel* is used to mean **Rel + pRel**

for any of the relations Rel for which pRel is defined. As an extension of this notation, $\{1\}^*$ denotes the disjunction of all linear-time relations together with all the seven relations of the form pRel.

CONDITION	RELATION	SYMBOL
$I_f < J_s$	I before J	$<$
$I_s > J_f$	I after J	$>$
$I_f = J_s$	I meets J	M
$I_s = J_f$	I met by J	$\mathrm{M}^{\smile}$
$I_s < J_s < I_f < J_f$	I overlaps J	O
$J_s < I_s < J_f < I_f$	I overlapped by J	$\mathrm{O}^{\smile}$
$I_s > J_s$, $I_f = J_f$	I finishes J	F
$I_s < J_s$, $I_f = J_f$	I finished by J	$\mathrm{F}^{\smile}$
$I_s > J_s$, $I_f < J_f$	I during J	D
$I_s < J_s$, $I_f > J_f$	I contains J	$\mathrm{D}^{\smile}$
$I_s = J_s$, $I_f < J_f$	I starts J	S
$I_s = J_s$, $I_f > J_f$	I started by J	$\mathrm{S}^{\smile}$
$I_s = J_s$, $I_f = J_f$	I equals J	$=$

Figure 2: **The Thirteen Linear-Time Interval Relations Between Two Intervals $I = [I_s,I_f]$ and $J = [J_s,J_f]$**

CONDITION	RELATION	SYMBOL
$p < I_s$, $p < J_s$	unrelated	U
$I_s = p < J_s$	adjacent	A
$I_s = J_s = p$	touching	T
$I_s < p < J_s$	partially precedes	p<
$I_s < J_s = p$	partially meets	pM
$I_s = J_s < p$	partially starts	pS
$I_s < J_s < p$	partially overlaps	pO
$J_s = p < I_s$	converse of adjacent	$\mathrm{A}^{\smile}$
$J_s < p < I_s$	converse of p<	$\mathrm{p}<^{\smile}$
$J_s < I_s = p$	converse of pM	$\mathrm{pM}^{\smile}$
$J_s < I_s < p$	converse of pO	$\mathrm{pO}^{\smile}$

Figure 3: **Relations Unique to Branching Time**

TABLE 1

Compositions of Branching Relations

	U	T	pS	A	pP	pM	pO	$A^{\smile}$	$pP^{\smile}$	$pM^{\smile}$	$pO^{\smile}$
U	I	U	[=]	U	U	U	U	U+[>]	U+[>]	U+$A^{\smile}$ +$pP^{\smile}$	U+$A^{\smile}$ +$pP^{\smile}$
T	U	[=]	T	[<]	A	A	A	$A^{\smile}$	$pP^{\smile}$	$pM^{\smile}$+$\alpha^{\smile}$	$pM^{\smile}$
pS	U	T	γ	A	[<]	[<]	pβ+α	$A^{\smile}$	$pP^{\smile}$	$pM^{\smile}$	$\alpha^{\smile}$
A	U+ [<]	A	A	A	A	A	A	T+A+ $A^{\smile}$+γ	$pP^{\smile}$+σ	σ	σ
pP	U+ [<]	pP	pP	pP	pP	pP	pP	pP+$\sigma^{\smile}$	{I}*	pλ+pρ	pλ+pρ
pM	U+A +pP	pM+α	pM	pP	pP	pP	pP	$\sigma^{\smile}$	pλ +p$\rho^{\smile}$	α+$\alpha^{\smile}$ +γ	pλ
pO	U+A +pP	pM	α	pP	pρ+α	pρ+α	α	$\sigma^{\smile}$	pλ +$\sigma^{\smile}$	pλ	α+$\alpha^{\smile}$ +γ
$A^{\smile}$	U	[>]	$A^{\smile}$	I	U	U	U	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	$pP^{\smile}$
$pP^{\smile}$	U	$A^{\smile}$	[>]	U	U+ {I}*	U	U	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	p$\rho^{\smile}$+ $\alpha^{\smile}$
$pM^{\smile}$	U	$A^{\smile}$	[>]	U	U	T+A+ $A^{\smile}$+γ	σ+p$\rho^{\smile}$ +$\alpha^{\smile}$	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	p$\rho^{\smile}$+ $\alpha^{\smile}$
$pO^{\smile}$	U	$A^{\smile}$	p$\rho^{\smile}$ +$\alpha^{\smile}$	U	U	$\sigma^{\smile}$+pρ +α	{I}*	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	p$\rho^{\smile}$+ $\alpha^{\smile}$

α : pO+{A_s<B_s} pρ : p(P+M+O) (A_s strictly first)
$\alpha^{\smile}$: pD+{A_s>B_s} σ : T+A+$pM^{\smile}$ (X=A_s)
γ : pS+{A_s=B_s} pλ : p(S+O+D) (X strictly last)

[>] : all relations in which A_s>B_s
[<] : all relations in which A_s<B_s
[=] : all relations in which A_s=B_s
{A_s>B_s}: all of Allen's relations in which A_s>B_s
{A_s<B_s}: all of Allen's relations in which A_s<B_s
{A_s=B_s}: all of Allen's relations in which A_s=B_s

TABLE 2

Compositions of Allen's and Branching Relations

	U	T	pS	A	pP	pM	pO	$A^{\smile}$	$pP^{\smile}$	$pM^{\smile}$	$pO^{\smile}$
S	U	T	S^*	A	P^*	pM+β	O^*+β	$A^{\smile}$	$pP^{\smile}$	$pM^{\smile}$	D^*
$S^{\smile}$	U	T	pS	A	pP	pM	pO	$A^{\smile}$	$pP^{\smile}$	$pM^{\smile}$	$pO^{\smile}$
P	$\nu^{\smile}$+P	P	P	P	P	P	P	$\sigma^{\smile}$+β	$p\rho^{\smile}$+pλ +[<f]	[<f]	[<f]
M	$\nu^{\smile}$	M	M	P	P	P	P	$\sigma^{\smile}$	$p\rho^{\smile}$+pλ	λ	λ
O	$\nu^{\smile}$	pM	O^*	pP	P^*	pM+β	O^*+β	$\sigma^{\smile}$	$p\rho^{\smile}$+pλ	pλ	λ^*
$F^{\smile}$	$\nu^{\smile}$	pM	pO	pP	pP	pM	pO	$\sigma^{\smile}$	$p\rho^{\smile}$+pλ	pλ	pλ
$D^{\smile}$	$\nu^{\smile}$	pM	pO	pP	pP	pM	pO	$\sigma^{\smile}$	$p\rho^{\smile}$+pλ	pλ	pλ
$P^{\smile}$	U	$A^{\smile}$	$pP^{\smile}$	U	U	$A^{\smile}$	$pP^{\smile}$	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	$pP^{\smile}$
$M^{\smile}$	U	$A^{\smile}$	$pP^{\smile}$	U	U	$A^{\smile}$	$pP^{\smile}$	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	$pP^{\smile}$
$O^{\smile}$	U	$A^{\smile}$	$pP^{\smile}$	U	U	$A^{\smile}$	$pP^{\smile}$	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	$pP^{\smile}$
F	U	$A^{\smile}$	$P^{\smile}$	U	$\nu^{\smile}$	$\sigma^{\smile}$	$p\rho^{\smile}$+pλ	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	$p\rho^{\smile}$
D	U	$A^{\smile}$	$p\rho^{\smile}$+D	U	$\nu^{\smile}$+P	$\sigma^{\smile}$+β	$\sigma^{\smile}$+pλ +[<f]	$A^{\smile}$	$pP^{\smile}$	$pP^{\smile}$	$p\rho^{\smile}$+D

where

A = (A_s,A_f) Is an interval starting at A_s and finishing at A_f.

pRel : If X is the branch point and $A_s < X < A_f$, then A pRel B ("A partially Rel B") means (A_s,X) Rel B. That is, the branch point is in A and the initial segment of A up to the branch point is in the relation Rel to B.

Rel* : Rel+pRel

X : The branch point

ν : U+$A^{\smile}$+$pP^{\smile}$ ($X<A_s$)

σ : T+A+$pM^{\smile}$ ($X=A_s$)

[<f] : P+M+O+S+D ($A_f<B_f$)

λ : S+O+D (=[<f]-β)

pλ : pS+pO+$pO^{\smile}$ (X strictly last)

ρ : P+M+O (Allen's Rels with $A_s<B_s$)

pρ : pP+pM+pO (A_s strictly first)

β : P+M (Allen's Rels with $A_f \leq B_s$)

TABLE 3

Compositions of Allen's Relations that Change in the Branching Time Model

	P	M	O	D	S
$P^{\smile}$	I	$A^{\smile}+\alpha^{\smile}+p\rho^{\smile}$	$\alpha^{\smile}+p\rho^{\smile}$	$\alpha^{\smile}+p\rho^{\smile}$	$\alpha^{\smile}+p\rho^{\smile}$
$M^{\smile}$	$A+\alpha+p\rho$	γ	$pM^{\smile}+\kappa^{\smile}$	$pM^{\smile}+\kappa^{\smile}$	$pM^{\smile}+\kappa^{\smile}$
$O^{\smile}$	$\alpha+p\rho$	$pM+\kappa$	$\gamma+\kappa+\kappa^{\smile}$	$\kappa^{\smile}$	$\kappa^{\smile}$
$D^{\smile}$	$\alpha+p\rho$	$pM+\kappa$	κ	$\gamma+\kappa+\kappa^{\smile}$	κ
$S^{\smile}$	$\alpha+p\rho$	$pM+\kappa$	κ	$\kappa^{\smile}$	[=s]

where
$A = (A_s, A_f)$ is an interval starting at A_s and finishing at A_f.
pRel : If X is the branch point and $A_s < X < A_f$, then A pRel B ("A partially Rel B") means (A_s, X) Rel B. That is, the branch point is in A and the initial segment of A up to the branch point is in the relation Rel to B.

Rel* : Rel+pRel
X : The branch point
τ : $D^{\smile}+F^{\smile}$
ρ : P+M+O (Allen's Rels with $A_s<B_s$ and $A_f<B_f$)
$p\rho$: pP+pM+pO (A_s strictly first)
β : P+M (Allen's Rels with $A_f \leq B_s$)
γ : $pS+\{=s\} = pS+S+S^{\smile}+1'$
[=s] : T+γ (All Rels with $A_s=B_s$)
κ : $pO+\{A_s<B_s<A_f\} = pO+O+D^{\smile}+F^{\smile}$
α : $pO+\{<s\} = pO+P+M+O+F^{\smile}+D^{\smile} = pO+\rho+\tau$

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